Today: Review

Next: Goodbye :(

Reminders:

Please fill out your CAPEs.

FINAL EXAM: TOMORROW 11:30am – 2:30pm

GH 242, PETER 108, YORK 2722

Seating / Room Assignment on Triton Ed

(8 points) 1. In each of the following examples, a vector space V is given, along with a subset S. Determine whether S is a subspace or not. In each case, explain why it is or is not a subspace.

(a) $V = M_{2\times 3}$ is the space of 2×3 matrices, A is a fixed 4×3 matrix, and $S \subseteq V$ is the set of 2×3 matrices X satisfying $AX^\top = 0$.

(b) $V = \mathbb{R}^3$, and $S = \left\{ \begin{bmatrix} 2s+3t \\ s-2t \\ -t \end{bmatrix} : s, t \in \mathbb{R} \right\}$.

(c) $V = \mathbb{R}^2$, and $S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : -1 \leq x + y \leq 1 \right\}$.

(d) $V = \mathbb{P}_4$ is the space of polynomials of degree ≤ 4, and S is the subset of polynomials p in V for which $p(-1) - 2p(0) + p(1) = 2018$.

Math 18: Lecture C	**Final Exam v. B**	**Name:** ______________
March 24, 2017	**(Total Points: *60*)**	**PID:** ______________

Instructions

1. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
2. **You may use tqo handwritten, double-sided pages of notes, but no books or other assistance during this exam.**
3. **Read each question carefully and answer each question completely.**
4. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
5. **Write your Name at the top of each page.**

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. (a) Consider the following vector equation:

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}.$$

Determine if this equation has a solution, and if so, describe it in parametric form.

(b) Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 1 & 1 & 3 & 1 \\ 0 & 2 & 2 & -2 \end{bmatrix}.$$

Describe the solution to the homogeneous equation $A\mathbf{x} = \mathbf{0}$ in parametric form.

(6 points) 2. Let A be the invertible matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 8 \\ 0 & 2 & -3 & -3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

(a) Determine the third column of the inverse matrix A^{-1}.

(b) Let $B = A^{\top}$, and let $C = B^3$. Calculate $\det(C^{-1})$.

(6 points) 3. For each of the following statement, circle **T** if it is *always* True; circle **F** if it is *ever* False. 2 points will be assigned for each correct response, 1 point for each blank non-response, and 0 points for each incorrect response. No justification is required.

(T F) If A and B are row equivalent matrices, then $\mathrm{Nul}(A) = \mathrm{Nul}(B)$.

(T F) The columns of a 4×25 matrix span $\mathbb{R}^4$.

(T F) The set of vectors $\mathbf{x} \in \mathbb{R}^3$ satisfying $x_1^2 + x_2^2 - x_3^2 = 0$ is a subspace of $\mathbb{R}^3$.

(9 points) 4. The matrix $A = \begin{bmatrix} 2 & 3 & 5 & 8 \\ 1 & 2 & 5 & 14 \\ -1 & -1 & 0 & 6 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 8 \end{bmatrix}$ has reduced row-echelon form $\mathrm{rref}(A) = \begin{bmatrix} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -15 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for $\mathrm{Col}(A)$, the column space of A.

(b) Find a basis for $\mathrm{Nul}(A)$, the nullspace of A.

(c) Find a basis for $\mathrm{Nul}(A)^{\perp}$, the orthogonal complement of $\mathrm{Nul}(A)$.

(9 points) 5. Consider the vectors $\mathbf{u}_1 = \begin{bmatrix}1\\1\\0\end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix}2\\-2\\1\end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix}-1\\1\\4\end{bmatrix}$, $\mathbf{v} = \begin{bmatrix}2\\3\\1\end{bmatrix}$.

(a) Show that $\mathcal{B} = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is an orthogonal basis for $\mathbb{R}^3$, and compute $\|\mathbf{u}_1\|$, $\|\mathbf{u}_2\|$, $\|\mathbf{u}_3\|$.

(b) Find the coordinates $[\mathbf{v}]_{\mathcal{B}}$ of the vector $\mathbf{v}$ in terms of the basis $\mathcal{B}$.

(c) Compute the orthogonal projection of $\mathbf{v}$ into the subspace spanned by the vectors $\{\mathbf{u}_2, \mathbf{u}_3\}$.

(8 points) 6. Let $A = \begin{bmatrix} 3 & 4 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. A has full rank.

(a) Use the Gram–Schmidt process on the rows of A to find an orthonormal basis for $\mathrm{Row}(A)$.

(b) Find an upper triangular matrix R and an orthogonal matrix Q such that $A^\top = QR$.

(8 points) 7. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

(a) Find all the eigenvalues of A, together with their algebraic multiplicities.

(b) Give a basis for the eigenspace corresponding to each eigenvalue of A.

(c) Is A diagonalizable? Explain why or why not.

(7 points) 8. Consider the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$.

(a) Explain how you can tell that A is diagonalizable without calculating any eigenvectors or eigenvalues.

(b) $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ are eigenvectors of A, with eigenvalues -1 and 2. Compute the inner product

$$\langle A^{2017}\mathbf{u}, \mathbf{v} \rangle.$$

(c) Compute the vector $\mathbf{v} + A^{2017}\mathbf{u}$.

Math 18 **Final Examination v. A** **Name:** ____________

March 22, 2017 **(Total Points: *50*)** **PID:** ____________

Instructions

1. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
2. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
3. **Read each question carefully and answer each question completely.**
4. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
5. **Write your Name at the top of each page.**

(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Consider the following matrix equation $A\mathbf{x} = \mathbf{b}$:

$$\begin{bmatrix} 1 & 2 & -1 \\ 1 & 3 & -3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(a) Determine the solution set of the matrix equation $A\mathbf{x} = \mathbf{b}$ and, if appropriate, write it in parametric form.

(b) Determine the solution set of the corresponding homogeneous matrix equation $A\mathbf{x} = \mathbf{0}$ and, if appropriate, write it in parametric form.

(6 points) 2. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Find A^{-1}, the inverse of A.

(b) Find the matrix X such that $A\,X = A^T$, the transpose of A.

(6 points) 3. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & -1 \end{bmatrix}$.

(a) Compute AB. Is AB an invertible matrix? Justify your answer.

(b) Compute BA. Is BA an invertible matrix? Justify your answer.

 Name: ____________________

(6 points) 4. The matrices $A = \begin{bmatrix} 3 & -1 & 1 & -6 \\ 2 & 1 & 9 & 1 \\ -3 & 2 & 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ are row equivalent.

(a) Find a basis for $\text{Col}(A)$, the column space of A.

(b) Find a basis for $\text{Nul}(A)$, the null space of A.

(c) Find a basis for $\text{Col}\left(A^T\right)^\perp$, the orthogonal complement of the column space of A^T. Be sure to explain how you know that it is a basis for $\text{Col}\left(A^T\right)^\perp$.

(6 points) 5. Let W be a subspace of $\mathbb{R}^n$ with an orthogonal basis $\mathcal{B}_W = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_p\}$, and let $\mathcal{B}_{W^\perp} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_q\}$ be an orthogonal basis for $W^\perp$, the orthogonal complement of W.

(a) Explain why $\mathcal{S} = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_p, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_q\}$ is an orthogonal set.

(b) Explain why the set $\mathcal{S}$ spans $\mathbb{R}^n$.

(c) Explain why $\mathcal{S}$ is linearly independent.

(d) Explain why $\dim(W) + \dim\left(W^\perp\right) = n$.

(6 points) 6. The set $\mathcal{B} = \left\{1,\ 1+2t,\ 1+2t+4t^2\right\}$ is a basis for $\mathbb{P}_2$, the vector space of polynomials of degree at most two. The polynomial $\mathbf{p} = 1 + 4t^2$. Find $\left[\mathbf{p}\right]_{\mathcal{B}}$, the coordinate vector for $\mathbf{p}$ with respect to the basis $\mathcal{B}$.

(6 points) 7. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 4 & 0 \\ 2 & 0 & 2 \end{bmatrix}$.

(a) Determine the eigenvalues of A. (Note: One of the eigenvalues of A is 0.)

(b) Find a matrix P that diagonalizes A. That is, find P so that $P^{-1}A\,P = D$, where D is a diagonal matrix.

(6 points) 8. Let $A = \begin{bmatrix} -1 & 2 & 10 \\ 2 & 1 & 10 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{bmatrix}$.

(a) Find an orthonormal basis for $\mathrm{Col}(A)$, the column space of A.

(b) Find an orthogonal matrix Q and an upper triangular matrix R such that $Q\,R = A$.