Today: §1.2: Row Reduction & Echelon Forms

& §1.3: Vector Equations

Next: §1.4: The Matrix Equation $A\underline{x} = \underline{b}$

Reminders:

MyMathLab Homework #1 & #2: Due Mon, Jan 22

MATLAB Homework #1: Due Fri Jan 19

Row Echelon Form

Reduced Row Echelon Form

$$
\left[\begin{array}{ccccccccc|c}
0 & \square & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & \square & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & \square & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & \square & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\xrightarrow{\text{row ops}}
\left[\begin{array}{ccccccccc|c}
0 & 1 & * & 0 & 0 & * & 0 & * & * & * \\
0 & 0 & 0 & 1 & 0 & * & 0 & * & * & * \\
0 & 0 & 0 & 0 & 1 & * & 0 & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

E.g. $\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$

Existence & Uniqueness

E.g. $\begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix} \xrightarrow{\text{row ops}} \begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

1.3

A vector (or column vector) is a list of real numbers in a column.

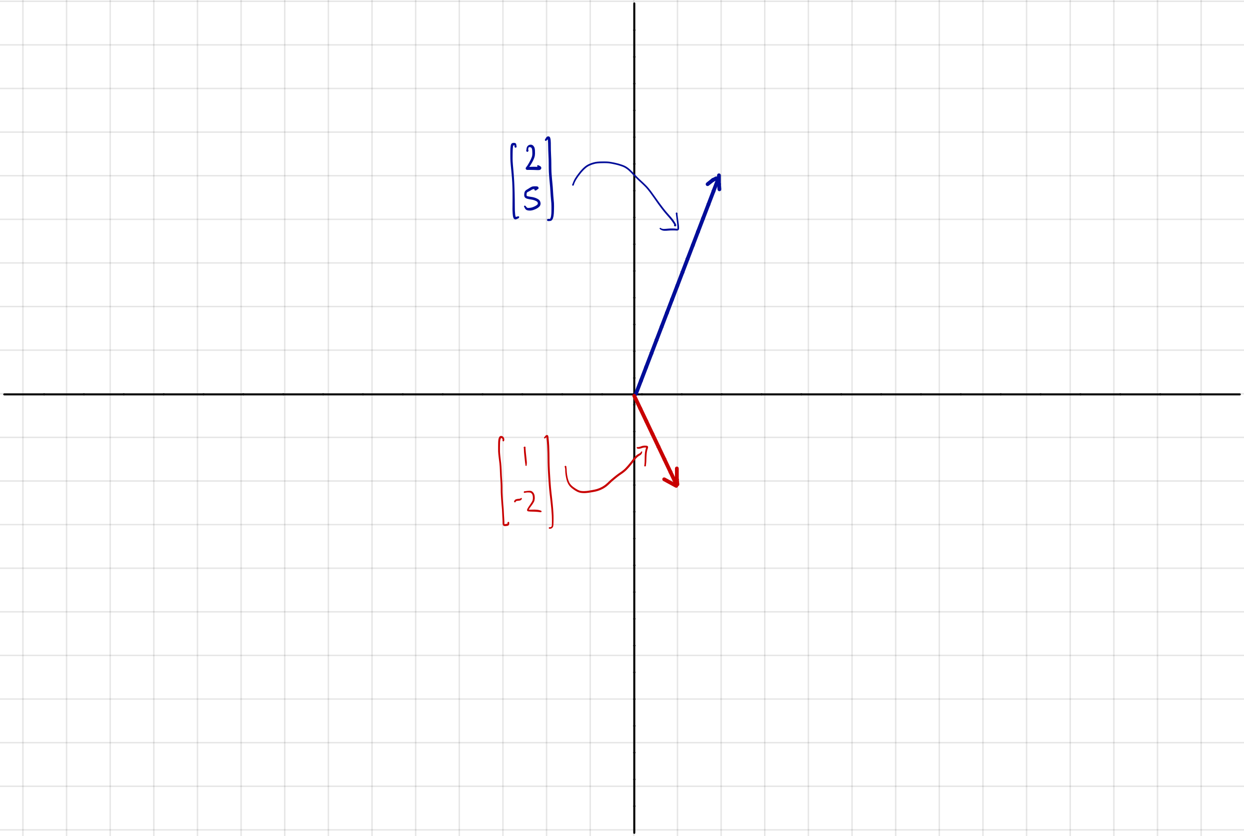
E.g. $\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \in \mathbb{R}^2$

E.g. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \in \mathbb{R}^4$

E.g. Today's closing stock prices for the stocks in the S&P 500 is a column vector in $\mathbb{R}$

Column vectors can be added and multiplied by scalars.

Geometry of vector addition and scalar multiplication.



Vector arithmetic works just like real number arithmetic.

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

$$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$$

$$\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$$

$$\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$$

$$c(\mathbf{u} + \mathbf{v}) \quad c\mathbf{u} + c\mathbf{v}$$

$$(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$$

$$c(d\mathbf{u}) = (cd)\mathbf{u}$$

$$1\mathbf{u} = \mathbf{u}$$

where

$$\mathbf{0} := \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

and

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{bmatrix}, \quad c\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} = \begin{bmatrix} cu_1 \\ cu_2 \\ \vdots \\ cu_n \end{bmatrix}$$

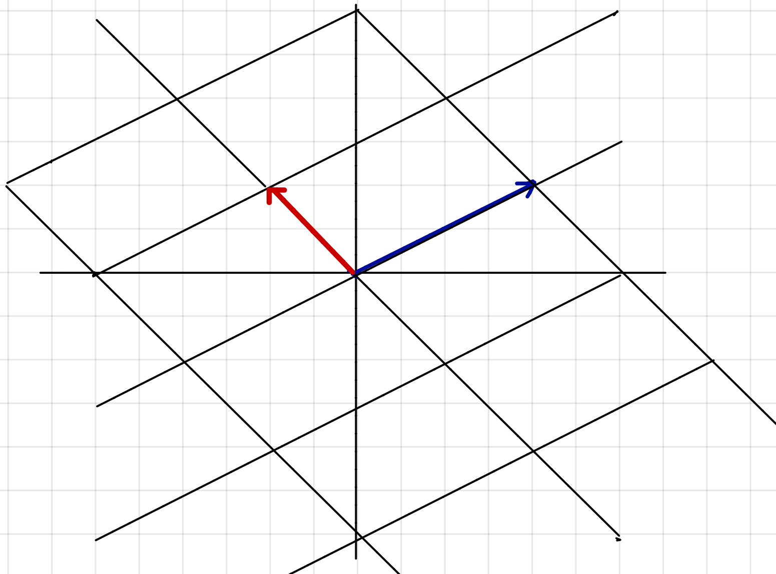
Linear Combinations

Given vectors $\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_n$ a linear combination is a vector of the form

$$x_1 \mathbf{u}_1 + x_2 \mathbf{u}_2 + \cdots + x_n \mathbf{u}_n$$

for some scalars $x_1, x_2, \ldots, x_n$.

Eg. $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$



Span

E.g. Is $\begin{bmatrix} 7 \\ 4 \\ 3 \end{bmatrix}$ in the span generated by $\left\{ \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \right\}$?