Today: § 1.7: Linear Independence

Next: § 1.8: Linear Transformations

Reminders:

MyMathLab Homework #1 & #2: Due TONIGHT!

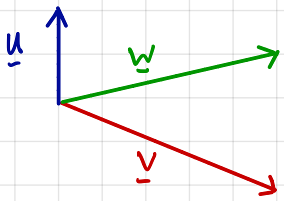
MATLAB Homework #1: Due TONIGHT!

MATLAB Homework #2: Due this Friday.

Midterm 1: Next Wed, Jan 31, 8-10pm.

Office hour: tomorrow (Tue) 9:30 - 11:00 am.

Consider the following three vectors.



How about these three vectors:

$$\underline{a} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 2 \end{bmatrix} \quad \underline{c} = \begin{bmatrix} 7 \\ -7 \\ 3 \\ 4 \end{bmatrix}$$

Definition. A collection of vectors $\{u_1, u_2, \dots, u_n\}$ is called **linearly dependent** if at least one of them is in the span of the others.

If a set of vectors is not linearly dependent, we call the vectors **linearly independent**.

Theorem. Vectors $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n$ are linearly independent if and only if the vector equation

$$x_1 \mathbf{u}_1 + x_2 \mathbf{u}_2 + \cdots + x_n \mathbf{u}_n = \mathbf{0}$$

has <u>only</u> the trivial solution $x_1 = x_2 = \cdots = x_n = 0$.

Corollary. The columns of a matrix A are linearly independent iff $A\underset{\sim}{x} = \underset{\sim}{0}$ has only the trivial solution $\underset{\sim}{x} = \underset{\sim}{0}$.

Eg.

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Eg.

$$\begin{bmatrix} 1 & 3 & 7 \\ 2 & -1 & -7 \\ 0 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

What if one of the vectors is $\mathbf{0}$?

What if there is only one vector ?

What if there are only two vectors?

E.g. $\begin{bmatrix} 3 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \end{bmatrix}$

E.g. $\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$

$$\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Theorem: The columns of a matrix A are linearly independent iff A has a pivot position in every column.
(Otherwise, the first non-pivotal column is a linear combination of the preceding ones, with coefficients read off of the rref.)

Corollary: If A has more columns than rows, its columns are linearly dependent.