Today: §1.8: Linear Transformations

Next: §1.9: The Matrix of a Lin. Transf.

Reminders:

MyMathLab Homework #3: Due Mon, Jan 29.

MATLAB Homework #2: Due this Friday.

Midterm 1: Next Wed, Jan 31, 8-10pm.

↳ practice midterms posted on webpage.

↳ seat assignment posted on TritonEd.

In previous math classes, you've studied functions.

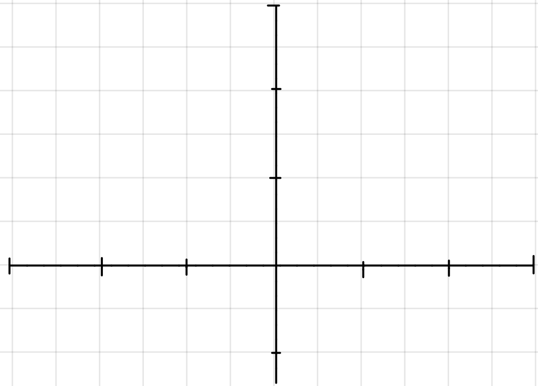
$$f : \underset{\text{domain}}{\mathbb{R}} \longrightarrow \underset{\text{codomain}}{\mathbb{R}}$$

Eg. $f(x) = 3x$. OR $g(x) = x^2 - 2x$. OR $h(x) = \sin(x)$

image: The image of 0 under g is $g(0) = 0$.

Is 0 the only thing in the domain whose image is 0?

range: The range of f is the set of all images of things in the domain.



An old-fashioned word for function is transformation.

Definition A matrix transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is a function given by matrix multiplication by some $m \times n$ matrix A.

E.g. $A = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \rightsquigarrow T: \mathbb{R} \longrightarrow \mathbb{R}$;

$$T\left(\quad \right)$$

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & -5 & -7 \\ -3 & 7 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The image of $\begin{bmatrix} 0 \\ 1/2 \\ -1/2 \end{bmatrix}$ under T is:

Does T map any other vector in $\mathbb{R}^3$ to this vector in $\mathbb{R}^2$?

What is the range of T?

Eg. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$. Is $\underline{b} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$ in the range of $T(\underline{x}) = A\underline{x}$?

$$\left[\begin{array}{cc|c} 1 & -3 & 3 \\ 3 & 5 & 2 \\ -1 & 7 & -5 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

What is the range of T?

Some geometric examples.

* $T(\underline{x}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \underline{x}$

* $T(\underline{x}) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \underline{x}$

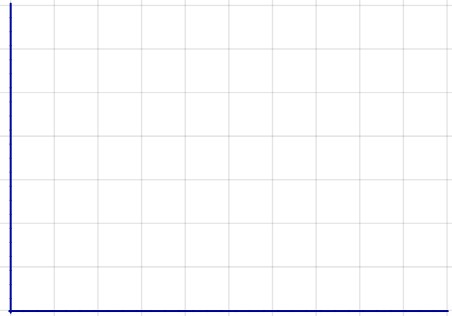
* $T(\underline{x}) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \underline{x}$

Linearity

Matrix multiplication behaves well under

addition: $A(\underline{u}+\underline{v}) =$

scalar multiplication: $A(c\underline{v})$



A linear transformation

$$T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

is a function with the properties

$T(\underline{u}+\underline{v}) =$ for all $\underline{u}, \underline{v} \in \mathbb{R}^n$

$T(c\underline{v}) =$ for all $\underline{v} \in \mathbb{R}^n$.

Matrix transformations are examples of linear transformations.

Properties.

$$T(\underline{0})$$

$$T(c_1\underline{v}_1 + c_2\underline{v}_2 + \cdots + c_k\underline{v}_k) =$$

$$c_1T(\underline{v}_1) + c_2T(\underline{v}_2) + \cdots + c_kT(\underline{v}_k)$$

E.g. $T: \mathbb{R}^3 \to \mathbb{R}^n$ linear.

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$