Today: §1.9: The Matrix of a Linear Transformation

Next: §2.1: Matrix Operations

Reminders:

MyMathLab Homework #3: Due Mon, Jan 29.

MATLAB Homework #2: Due TONIGHT

Midterm 1: Next Wed, Jan 31, 8-10pm.

↳ practice midterms posted on webpage.

↳ seat assignment posted on TritonEd.

A function/transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is called linear if it respects addition and scalar multiplication.

$$T(\underline{u}+\underline{v}) = T(\underline{u}) + T(\underline{v})$$

$$T(c\underline{u}) = cT(\underline{u})$$

E.g.

$$\Rightarrow \quad T(\underline{0}) =$$

$$T(c_1\underline{v}_1 + c_2\underline{v}_2 + \cdots + c_k\underline{v}_k) =$$

$$\therefore \quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)$$

$$T: \mathbb{R}^3 \to \mathbb{R}^n$$

Definition The standard basis vectors in $\mathbb{R}^n$ are

$$\underline{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \underline{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad \cdots, \quad e_n = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

E.g. In $\mathbb{R}^2$, there are 2 standard basis vectors

In $\mathbb{R}^3$, there are 3

They are the columns of the identity matrix

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & & 1 \end{bmatrix}$$

Theorem. A linear transformation T is completely determined by its image on the standard basis vectors

$$T(\underline{e}_1),\ T(\underline{e}_2),\ \dots,\ T(\underline{e}_n). \qquad T: \mathbb{R}^n \to \mathbb{R}^m$$

Moreover, every linear transformation is a matrix transformation!

Eg. The "identity" function $T(\underline{x}) = \underline{x}$ is linear.
What is its matrix?

Eg. The rotation ccw 45° is linear. (Why?)
What is its matrix?

More function language.

Definition: A transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is called

onto

one-to-one

E.g. Every rotation is both one-to-one and onto.

E.g. $T(\underline{x}) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \underline{x}$

E.g. $T(\underline{x}) = \begin{bmatrix} 3 & 1 \\ 5 & 7 \\ 1 & 3 \end{bmatrix} \underline{x}$

<u>Theorem</u>: Let $T: \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation with standard matrix A.

(a) T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ iff :

(b) T is one-to-one iff :