

Today: § 2.1: Matrix Operations

Next: Midterm 1 Review

Reminders:

MyMathLab Homework #3: Due **TONIGHT**.

Midterm 1: **THIS** Wed, Jan 31, 8-10pm.

↳ covers § 1.1-1.5, 1.7-1.9

↳ practice midterms posted on webpage.

↳ seat assignment posted on TritonEd.

## § 2.1: Matrix Operations

We can treat matrices (of the same size!) like vectors, and add them and multiply them by scalars.

$$\text{Eg. } A = \begin{bmatrix} 1 & -1 & 7 \\ 3 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 2 & 1 \\ 4 & 2 & 8 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$$

$$2A - B$$

$$B + 2C$$

Addition and scalar multiplication of matrices behave nicely together, just like for real numbers:

$$A + B$$

$$r(A+B)$$

$$(A+B) + C$$

$$(r+s)A$$

$$A + O$$

$$r(sA)$$



zero matrix  
of appropriate  
dimensions

Matrix addition and scalar multiplication

correspond to addition and scalar multiplication  
of linear transformations.

Eg.  $A = \begin{bmatrix} 1 & -1 & 7 \\ 3 & 2 & 1 \end{bmatrix}$        $B = \begin{bmatrix} 0 & 2 & 1 \\ 4 & 2 & 8 \end{bmatrix}$

$$T(\underline{x}) = A\underline{x}$$

$$S(\underline{x}) = B\underline{x}$$

$$\underline{x} \in \mathbb{R}^3$$

$$(T+S)(\underline{x}) =$$

$$(rT)(\underline{x}) =$$

# Matrix Multiplication

If  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  and  $S: \mathbb{R}^m \rightarrow \mathbb{R}^k$

we can compose them

$$\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m \xrightarrow{S} \mathbb{R}^k$$

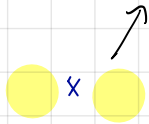
$S \circ T: \mathbb{R}^n \rightarrow \mathbb{R}^k$ . Notice:

$$S \circ T(\underline{u} + \underline{v}) =$$

$$S \circ T(r \underline{v}) =$$

$$T: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$T(\underline{x}) = A\underline{x}$$

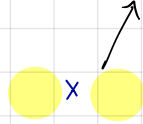


$$A = \left[ \begin{array}{c} \\ \\ \end{array} \right]$$

$$S \circ T(\underline{x}) = S(T(\underline{x})) =$$

$$S: \mathbb{R}^m \rightarrow \mathbb{R}^k$$

$$S(\underline{y}) = B\underline{y}$$



Definition: If  $A$  is  $m \times n$  and  $B$  is  $k \times m$   
the product  $BA$  is defined by

$$BA = B \begin{bmatrix} \underline{a_1} & \underline{a_2} & \dots & \underline{a_n} \end{bmatrix} = \begin{bmatrix} B\underline{a_1} & B\underline{a_2} & \dots & B\underline{a_n} \end{bmatrix}.$$

Eg.  $\begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

Eg.  $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}$

# Properties of Matrix Multiplication

$$A(BC) = (AB)C$$

$$A(B+C) = AB + AC$$

$$(B+C)A = BA + CA$$

$$(rA)B = r(AB) = A(rB)$$

$$\text{If } A \text{ is } m \times n, \quad I_m A = A = A I_n$$

$n \times n$   
identity  
matrix  
(MATLAB: `eye(n)`)



Even square matrices don't generally commute.

$$A = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$B = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$AB =$$

$$BA =$$

# Transpose

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$$

$$\text{Eg. } \begin{bmatrix} -5 & 1 & 0 \\ 2 & -3 & 4 \\ 0 & 1 & 3 \end{bmatrix}^T =$$

$$; \begin{bmatrix} 1 & 7 & 6 \\ 7 & 3 & -1 \\ 6 & -1 & 4 \end{bmatrix}^T =$$

## Properties

If  $A$  is  $m \times n$ , then  $A^T$  is  $n \times m$

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$(AB)^T = B^T A^T$$

$$(rA)^T = rA^T$$