Today: §2.1: Matrix Operations

Next: Midterm 1 Review

Reminders:

MyMathLab Homework #3: Due TONIGHT.

Midterm 1: THIS Wed, Jan 31, 8-10pm.

↳ covers §1.1-1.5, 1.7-1.9

↳ practice midterms posted on webpage.

↳ seat assignment posted on TritonEd.

§ 2.1 : Matrix Operations

We can treat matrices (of the same size!) like vectors, and add them and multiply them by scalars.

Eg. $A = \begin{bmatrix} 1 & -1 & 7 \\ 3 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2 & 1 \\ 4 & 2 & 8 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$

$2A - B$

$B + 2C$

Addition and scalar multiplication of matrices behave nicely together, just like for real numbers:

$A + B$

$(A+B) + C$

$A + O$

zero matrix of appropriate dimensions

$r(A+B)$

$(r+s)A$

$r(sA)$

Matrix addition and scalar multiplication correspond to addition and scalar multiplication of linear transformations.

Eg. $A = \begin{bmatrix} 1 & -1 & 7 \\ 3 & 2 & 1 \end{bmatrix}$ $\qquad B = \begin{bmatrix} 0 & 2 & 1 \\ 4 & 2 & 8 \end{bmatrix}$

$T(\underline{x}) = A\underline{x}$ $\qquad S(\underline{x}) = B\underline{x}$ $\qquad \underline{x} \in \mathbb{R}$

$(T+S)(\underline{x}) =$

$(rT)(\underline{x}) =$

Matrix Multiplication

If $T : \mathbb{R}^n \to \mathbb{R}^m$ and $S : \mathbb{R}^m \to \mathbb{R}^k$

we can compose them

$$\mathbb{R}^n \xrightarrow{T} \mathbb{R}^m \xrightarrow{S} \mathbb{R}^k$$

$S \circ T : \mathbb{R}^n \to \mathbb{R}^k$. Notice:

$S \circ T(\underline{u} + \underline{v}) =$

$S \circ T(r\underline{v}) =$

$T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ $\qquad$ $S : \mathbb{R}^m \longrightarrow \mathbb{R}^k$

$T(\underline{x}) = A\underline{x}$ $\qquad$ $S(\underline{y}) = B\underline{y}$

$\times$ $\qquad$ $\times$

$A = \begin{bmatrix} \quad \end{bmatrix}$

$S \circ T(\underline{x}) = S(T(\underline{x})) =$

Definition: If A is $m \times n$ and B is $k \times m$ the product BA is defined by

$$BA = B\begin{bmatrix} \underline{a}_1 & \underline{a}_2 & \cdots & \underline{a}_n \end{bmatrix} := \begin{bmatrix} B\underline{a}_1 & B\underline{a}_2 & \cdots & B\underline{a}_n \end{bmatrix}.$$

E.g. $\begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

E.g. $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}$

Properties of Matrix Multiplication

$$A(BC) = (AB)C$$

$$A(B+C) = AB + AC$$

$$(B+C)A = BA + CA$$

$$(rA)B = r(AB) = A(rB)$$

If A is $m \times n$, $I_m A = A = A I_n$

$n \times n$ identity matrix (MATLAB: eye(n))

Even square matrices don't generally commute.

$A = \begin{bmatrix} & \\ & \end{bmatrix}$ $B = \begin{bmatrix} & \\ & \end{bmatrix}$

$AB =$

$BA =$

Transpose

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$$

Eg. $\begin{bmatrix} -5 & 1 & 0 \\ 2 & -3 & 4 \\ 0 & 1 & 3 \end{bmatrix}^T =$; $\begin{bmatrix} 1 & 7 & 6 \\ 7 & 3 & -1 \\ 6 & -1 & 4 \end{bmatrix}^T =$

Properties

If A is $m \times n$, then A^T is $n \times m$

$(A^T)^T = A$ $\qquad$ $(A+B)^T = A^T + B^T$

$(AB)^T = B^T A^T$ $\qquad$ $(rA)^T = rA^T$