1. Let $X$ and $Y$ be continuous random variables with joint density function

$$f_{X,Y}(x, y) = \begin{cases} \frac{x}{2} + \frac{y}{4}, & 0 \leq x \leq 1, \ 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) (5 points) Compute $P(X > 2Y)$.
(b) (5 points) What is the marginal density $f_X$ of $X$?
(c) (5 points) Are $X$ and $Y$ independent? Explain.

2. An urn contains 5 balls: two balls labeled 1, two balls labeled 2, and one ball labeled 3. You sample 2 balls from the bin, without replacement; the number on the first one is $X$, and the number on the second one is $Y$.

(a) (5 points) Compute the joint probability mass function of $X$ and $Y$. (You may find it convenient to express it in the form of a chart.)
(b) (5 points) Compute the probability mass function of $X$ and the probability mass function of $Y$.
(c) (5 points) Are $X$ and $Y$ independent? Justify your answer.

3. Let $U_1, U_2, \ldots, U_n, \ldots$ be independent, identically distributed random variables, each with the Uniform$[-2, 2]$ distribution. Let $S_n = U_1 + U_2 + \cdots + U_n$.

(a) (5 points) Compute $\mathbb{E}(S_n)$ and $\text{Var}(S_n)$.
(b) (5 points) For any $\epsilon > 0$, what can you say about

$$\lim_{n \to \infty} \mathbb{P}\left( \frac{|S_n|}{n^{2/3}} \geq \epsilon \right)$$

4. Let $T$ be the triangle in $\mathbb{R}^2$ with vertices $(0, 0), (0, 1),$ and $(1, 1)$ (including the interior). Suppose that $P = (X, Y)$ is a point chosen uniformly at random inside of $T$.

(a) (5 points) What is the joint density function of $(X, Y)$? Use this to compute $\text{Cov}(X, Y)$.
(b) (5 points) Determine if $X$ and $Y$ are independent.

5. Suppose $X_1, X_2, \ldots, X_n, \ldots$ are i.i.d. random variables with mean $\mathbb{E}(X_j) = 0$ and variance $\text{Var}(X_j) = 1$. Determine the following limits with precise justifications.

(a) (5 points) $\lim_{n \to \infty} \mathbb{P}\left( -\frac{n}{4} \leq X_1 + \cdots + X_n < \frac{n}{2} \right)$
(b) (5 points) $\lim_{n \to \infty} \mathbb{P}(X_1 + \cdots + X_n = 0)$