1. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space.
   (a) Suppose that $A, B \in \mathcal{F}$ satisfy
   \[ \mathbb{P}(A) + \mathbb{P}(B) > 1. \]
   Making no further assumptions on $A$ and $B$, prove that $A \cap B \neq \emptyset$.

   (b) Prove that $A$ is independent from itself if and only if $\mathbb{P}(A) \in \{0, 1\}$. 
2. Roll two fair dice repeatedly. If the sum is $\geq 10$, then you win.
   
   (a) What is the probability that you start by winning 3 times in a row?

   (b) What is the probability that after rolling the pair of dice 5 times you win exactly 3 times?

   (c) What is the probability that the first time you win is before the tenth roll (of the pair), but after the fifth?
3. A box contains 3 coins, two of which are fair and the third has probability $3/4$ of coming up heads. A coin is chosen randomly from the box and tossed 3 times.

(a) What is the probability that all 3 tosses are heads?

(b) Given that all three tosses are heads, what is the probability that the biased coin was chosen?
4. Let $X$ be a discrete random variable taking the values $\{1, 2, \ldots, n\}$ all with equal probability. Let $Y$ be another discrete random variable taking values in $\{1, 2, \ldots, n\}$. Assume that $X$ and $Y$ are independent. Show that $P(X = Y) = \frac{1}{n}$. (Hint: you do not need to know the distribution of $Y$ to calculate this.)
5. Consider a point $P = (X, Y)$ chosen uniformly at random inside of the triangle in $\mathbb{R}^2$ that has vertices $(1, 0)$, $(0, 1)$, and $(0, 0)$. Let $Z = \max(X, Y)$ be the random variable defined as the maximum of the two coordinates of the point. For example, if $P = \left(\frac{1}{2}, \frac{1}{3}\right)$, then $Z = \max(X, Y) = \frac{1}{2}$. Determine the cumulative distribution function of $Z$. Determine if $Z$ is a continuous random variable, a discrete random variable, or neither. If continuous, determine the probability density function of $Z$. If discrete, determine the probability mass function of $Z$. If neither, explain why.

(Hint: Draw a picture.)