Math 182: Hidden Data in Random Matrices

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TODAY: Histograms and Linear Statistics

NEXT: The Trace and the Large-N Limit

HW 2:
Lab 2: } Regrade Window Wed Feb 12 8am → Fri Feb 14 8pm
Histograms & Empirical Laws

A histogram $h_{\Pi, \lambda}$ of a point set $\lambda$ encodes the histogram statistics

$$h_{[a,b]}(\lambda) = \frac{1}{N} \# \{ j : \lambda_j \in [a,b] \}$$

$h_{\Pi, \lambda}$ is an "approximate density" for the empirical random variable

$E_\lambda : \mathbb{P}(E_\lambda = x) = \ldots$
Histograms vs. Polynomial Linear Statistics

\[ h_{[a,b]}(\lambda) = \frac{1}{n} \sum_{j=1}^{n} \mathbb{I}_{[a,b]}(\lambda_j) \]

Example of a linear statistic

\[ \frac{1}{2} \sum_{j=1}^{n} w(\lambda_j) \]

“weight function”

Hard to compute with weight function \( w = \mathbb{I}_{[a,b]} \).

More familiar examples:
Approximating with Polynomials

It will turn out to be possible to really compute linear statistics of eigenvalues, with a polynomial weight function \( w \). We'd really like to do it with \( w = \mathbb{1}_{[a,b]} \). Fortunately, we can approximate \( \mathbb{1}_{[a,b]} \) with polynomials.

**Theorem (Bernstein, 1920s)**

Let \([a,\beta]\) be a closed interval, and let \( w: [a,\beta] \to \mathbb{R} \). There is a sequence \( B_n \) of polynomials (\( \deg P_n = n \)) such that

\[
\lim_{n \to \infty} B_n(x) = w(x) \quad \text{for all } x \in [a,\beta].
\]

s.t. \( w \) is continuous at \( x \).
Proof of Bernstein’s Theorem

Step 1: reduce to \([a, \beta] = [0, 1]\).

\[ x \]

\[ \frac{x - a}{\beta - a} \]

Step 2: the "Probabilistic Method"