Turn in the homework by 1:00pm in lecture on Monday. Late homework will not be accepted.


2. Exercise 11.12 on p. 278 in Lee.


4. Problem 11-7 on p. 300 in Lee.

5. Problem 11-9 on p. 301 in Lee.


8. Let $M$ be a smooth manifold, and let $X, Y \in \mathcal{X}(M)$ be smooth vector fields. Complete the following simpler proof that the Lie derivative $\mathcal{L}_Y(X) = [Y, X]$. Let $\theta$ be the flow of $Y$, and for fixed $p \in M$ and all sufficiently small $t, h \in \mathbb{R}$, define

$$\alpha(s, t) = X_{\theta_s(p)}(f \circ \theta_{-t}).$$

Note that $\alpha$ is a map from an open set in $\mathbb{R}^2$ into $\mathbb{R}$. Show that

$$\frac{\partial \alpha}{\partial s}(0, 0) = -X_p(Yf), \quad \frac{\partial \alpha}{\partial t}(0, 0) = Y_p(Xf).$$

Now calculate $\frac{d}{dt}|_{t=0} \alpha(t, t)$ to complete the proof.

9. Even though polar coordinates $(r, \theta)$ are not globally smooth on $\mathbb{R}^2 \setminus \{0\}$, show that there is a global differential $d\theta$: let $\omega$ be the following smooth covariant vector field on $\mathbb{R}^2 \setminus \{0\}$:

$$\omega = \frac{xdy - ydx}{x^2 + y^2}.$$

Define $P: (0, \infty) \times \mathbb{R} \rightarrow \mathbb{R}^2 \setminus \{0\}$ by $P(r, \theta) = (r \cos \theta, r \sin \theta)$. Show that $P^* \omega = d\theta$. 
