Turn in the homework by 1:00pm in lecture on Monday. Late homework will not be accepted.

1. Exercise 1.1 on p. 3 in Lee.

2. Exercise 1.7 on p. 7 in Lee.


4. Problem 1-8 on p. 31 in Lee.

5. Exercise 2.7 on p. 35 in Lee.


8. Problem 2-7 on p. 48 in Lee.


For problems 11 and 12, we use the nominally different definition of chart and atlas given in Proposition 1.20 in the course notes. Ergo, problem 11 is to prove that proposition.

11. Prove Proposition 1.20 and Remark 1.21 in the course notes.

For the final problem, we use the following notation: if $E$ is a subspace of $\mathbb{R}^n$, then $E^\perp$ is its orthogonal complement: the set of all vectors $v \in \mathbb{R}^n$ with the property that $v \cdot w = 0$ for all $w \in E$. For any two real vector spaces $E, F$, $\text{Hom}(E, F)$ denotes the set of all linear maps from $E$ to $F$. If $E$ and $F$ are Hilbert spaces, $\text{Hom}(E, F)$ is also a Hilbert space, with the Fröbenius inner product $\langle A, B \rangle = \text{Tr}(B^*A))$.

12. The Grassmannian of $k$-planes in $\mathbb{R}^n$, denoted $\text{Gr}(n, k)$, is the set of all $k$-dimensional subspaces of $\mathbb{R}^n$. For $E \in \text{Gr}(n, k)$, define $U_E = \{ F \in \text{Gr}(n, k) : F \cap E^\perp = \{0\} \}$.

   (a) Show that every $F \in U_E$ is the graph of a unique linear map $T_F : E \to E^\perp$. 
(b) Define $\varphi_E : U_E \to \text{Hom}(E, E^\perp)$ by $\varphi_E(F) = T_F$. Show that the collection

$$\{(U_E, \varphi_E, \text{Hom}(E, E^\perp)) : E \in \text{Gr}(n, k)\}$$

is a smooth atlas on $\text{Gr}(n, k)$. What is the dimension of this manifold?

(c) Carefully explain how this atlas, in the case $\text{Gr}(n + 1, 1) = \mathbb{P}^n$, relates to the one in Example 1.9 in the course notes.

Note: parts (a) and (b) of this problem are essentially solved in Example 1.36 in Lee. You can read it for guidance, but you still won’t really understand how it works until you’ve done it yourself (thus, this problem).