Math 251B: “Winter” 2019
Homework 2

Available Wednesday, February 13 || Due Friday, March 1

Turn in the homework in lecture by 1pm. Late homework will not be accepted.

1. Problem 20-1 on p. 536 in Lee.


5. Problem 20-8 on p. 537 in Lee.


9. Show that the following matrices form a basis for the Lie algebra $su(2)$:

$$E_1 = \frac{1}{2} \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}, \quad E_2 = \frac{1}{2} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad E_3 = \frac{1}{2} \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}.$$

Compute the brackets $[E_1, E_2]$, $[E_2, E_3]$, and $[E_3, E_1]$. Show that there is a linear isomorphism $\varphi: su(2) \to \mathbb{R}^3$ so that $\varphi([X, Y]) = \varphi(X) \times \varphi(Y)$ for all $X, Y \in su(2)$. (Here $\times$ denotes the cross product on $\mathbb{R}^3$.)

10. Show that the Lie algebras $su(2)$ and $so(3)$ are isomorphic. Nevertheless, show that the corresponding Lie groups $SU(2)$ and $SO(3)$ are not isomorphic. [Hint: Show that $SU(2)$ is connected.]