* Video Lectures: 0, 1.1, 1.2 posted on YouTube

* Quiz 1: Thursday, October 8, 1-1:50pm or 7-7:50pm
  - Sign up for your quiz time: Google form
  - The quiz will be posted on Gradescope; you will turn it in on Gradescope.

* HW 1: Due Monday, October 12, 9 pm.
Question: Suppose $A \subseteq 2^\Omega$ satisfies:

* (1) $\Omega \in A$
* (2) $A$ is closed under $\Delta$
* (3) $A$ is closed under $\cap$

$\Delta A B = A \setminus B \cup B \setminus A$
$(A \cap B^c) \cup (B \setminus A^c)$

Is $A$ a field? Yes.

Complement: is $A^c = A \Delta ?$
$\Omega \Delta A = (\Omega \cap A^c) \cup (A \cap \Omega^c) = A^c$
Open set $U \subset \mathbb{R}^d$

$$U = \bigcup_{j=1}^{\infty} B(x_j, r_j)$$

$$B(x_j, r_j) = \bigcup_{i=1}^{\infty} B(y_i, r_i)$$

$$B(y_j, r_j) = B\left(\frac{\alpha_j + \beta_j}{2}, \frac{\beta_j - \alpha_j}{2}\right) = (\alpha_j, \beta_j)$$

\[ a < \alpha_j < \beta_j < b \]
\[ |a - \alpha_j|, |\beta_j - b| < \frac{1}{n} \]
$U \subseteq \mathbb{R}^d$

**Claim:**

$$U = \bigcup_{x \in U} B(x, r)$$

**Proof:**

1. **Openness of $U$:**
   - For each $x \in U$, choose $r > 0$ such that $B(x, r) \subseteq U$.

2. **Construction:**
   - Since $U$ is open, for each $x \in U$, there exists an open ball $B(x, r) \subseteq U$.

3. **Union:**
   - Thus, $U = \bigcup_{x \in U} B(x, r)$.
\[ \mathcal{F} = \{ F : F \text{ is a flag} \} \]

is the "smallest flag"

\[ \mathcal{F}_0 \]

is in this list.

The set of flags is closed under \( \bigcup \) arbitrary union.

If \( F \) is any flag, then \( \mathcal{F}_0 \subseteq \mathcal{F} \).

[flag = \( \sigma \)-field over \( \mathbb{D} \), containing \( \mathcal{E} \)]

\[ \mathcal{F}_0 = \bigcap \{ \text{all flags} \} \]