Turn in the homework by 11:00am in lecture on Wednesday. Late homework will not be accepted.

1. Exercise 4.1 on pp. 90-91 in Chung & Williams.
2. Exercise 4.2 on p. 91 in Chung & Williams.
3. Exercise 5.1 on p. 112 in Chung & Williams.
4. Exercise 5.2 on p. 113 in Chung & Williams.
5. Exercise 5.6 on p. 113 in Chung & Williams.

6. Let $M$ be a local martingale. Suppose that, for each $t \in \mathbb{R}^+$, the family of processes
\[
\{M^\tau : \tau \text{ is a stopping time}\}
\]
is uniformly integrable. Prove that $M$ is actually a martingale.

7. Let $M$ be an $L^2$ martingale. If $X, Y \in \Lambda^2(\mathcal{P}, M)$ are $\mu_M$-equivalent, i.e.
\[
\mu_M\{(t, \omega) : X(t, \omega) \neq Y(t, \omega)\} = 0
\]
then the two martingales $\int_0^t X \, dM$ and $\int_0^t Y \, dM$ are indistinguishable.

8. Let $(\mathcal{F}_t)_{t \geq 0}$ be a standard filtration, and let $M = (M_t)_{t \geq 0}$ be an adapted, $L^2$, right continuous process with $M_0 = 0$, $E(M_t) = 0$ for all $t \geq 0$, and with independent increments: $M_t - M_s$ is independent from $\mathcal{F}_s$ for $0 \leq s < t < \infty$. Prove that $M$ is an $L^2$-martingale, that $A_t = E(M_t^2)$ is a right-continuous increasing (deterministic) process, and that $N_t = M_t^2 - A_t$ is a martingale.

9. Let $B$ be a Brownian motion in $\mathbb{R}$. Let $f \in L^2(\mathbb{R}^+)$
   (a) Show that the (deterministic) process $f$ is in $\Lambda^2(\mathcal{P}, B)$.
   (b) Show that $\lim_{t \to \infty} \int_0^t f(s) \, dB_s$ exists in $L^2$. Does it also exist a.s.?
   (c) Let $g$ be a measurable function on $\mathbb{R}^+$ such that $\int_0^t g(s)^2 \, ds < \infty$ for each $t \in \mathbb{R}^+$.
      Show that
      \[
      X_t = \exp \left\{ \int_0^t g(s) \, dB_s - \frac{1}{2} \int_0^t g(s)^2 \, ds \right\}
      \]
is an $L^2$ martingale, and compute $E(X_t^2)$. If $g \in L^2(\mathbb{R}^+)$, show that $X$ is an $L^2$-bounded martingale.