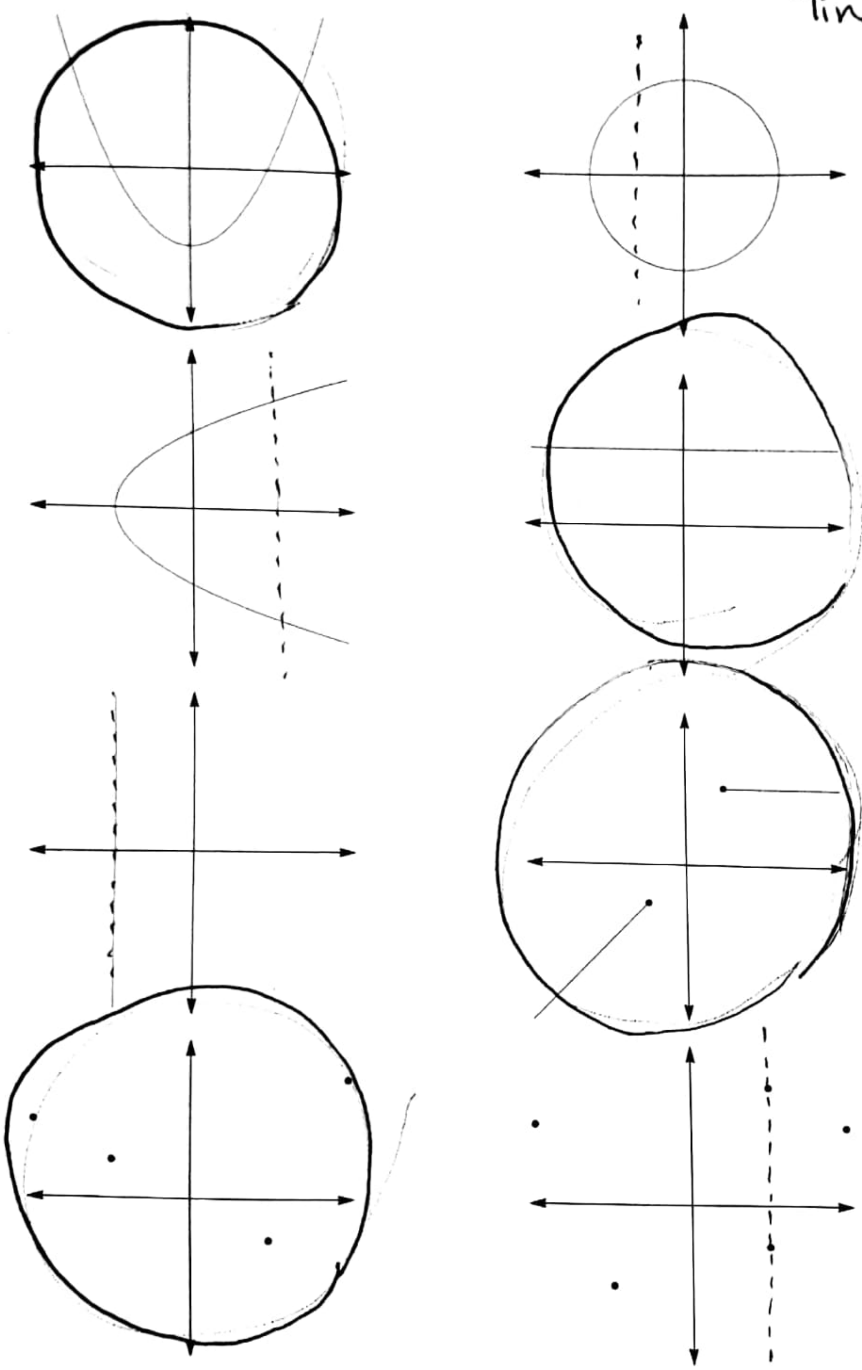
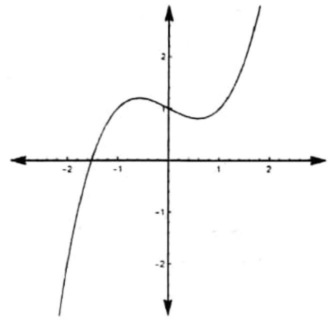
Question 1. (8 points) Which of the following are the graph of a function? *Circle all that apply.*

↳ passes vertical line test

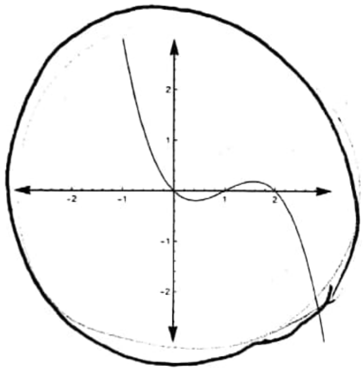


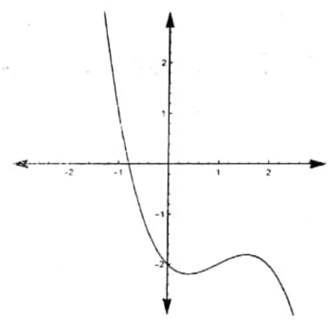
Question 2. (4 points) The graph of a function $h(x)$ is given by:

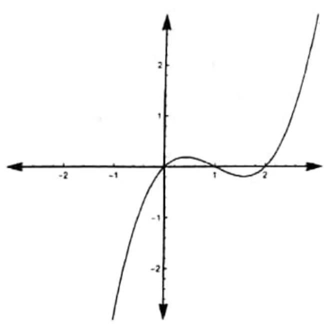


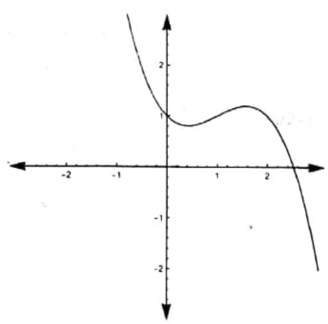
- flip across x-axis
- shift up one unit
- shift right one unit

Which of the following could be the graph of $g(x) = -h(x-1)+1$? *Circle one.*

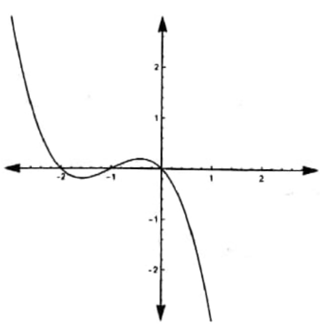


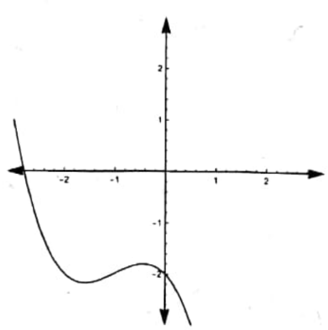






most common incorrect answer





Question 3. (5 points) Which of the following are equal to the function $f(x) = 3x + 1$ with the domain of all real numbers? *Circle all that apply.*

(A) $g_1(x) = |3x - 1|$ with the domain of all real numbers $\left\{ \begin{array}{l} \text{absolute value does } \underline{\text{not}} \text{ mean} \\ \text{"take away the minus signs"} \end{array} \right.$

(B) $g_2(x) = 3x + 3 - 2$ with the domain of all real numbers → simplify

(C) $g_3(x) = \begin{cases} 3x+1 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ f and g_3 are equal at all x-values in the domain, so they are the same. Note: $f(0)=1=g_3(0)$

(D) $g_4(x) = \begin{cases} 3x+1 & \text{if } x \neq 0 \\ 2 & \text{if } x = 0 \end{cases}$ $f(0) \neq g_4(0)$, so they are not the same function.

(E) $g_5(x) = 3x + 1$ with the domain $[0, \infty)$

2 functions with different domains are always different functions, even if they have the same formula

Question 4.

a) (3 points) What is the largest domain for the function $g(x) = \sqrt{2x+1} + 3$ such that it is defined and produces a real number? *Write your answer in interval notation.*

Squareroot is not defined for negative numbers, so for x to be in the domain it must satisfy

$$2x+1 \geq 0$$
$$2x \geq -1$$
$$x \geq -\frac{1}{2}$$

Interval notation: $\boxed{[-\frac{1}{2}, \infty)}$

b) (3 points) What is the range of g corresponding to this domain? *Write your answer in interval notation.*

$\sqrt{2x+1}$ can take on any value ≥ 0 (it is equal to 0 at $x=-\frac{1}{2}$), so if we add 3, $g(x)=\sqrt{2x+1}+3$ can take on any value ≥ 3.

Interval notation: $\boxed{[3, \infty)}$

Question 5.

a) (6 points) What real values of x satisfy the inequality $|x+2| < 3$? *Write your answer in interval notation.*

$$-3 < x+2 < 3$$

$$-5 < x < 1$$

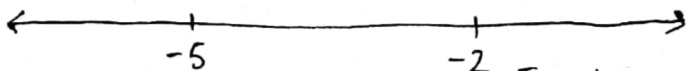
strict inequality, so endpoints not included

Interval notation: $\boxed{(-5, 1)}$

b) (10 points) What real values of x satisfy the following equation? Split into regions:

$$|x+5| + |x+2| = 9$$

$$|b| = \begin{cases} b & \text{if } b \geq 0 \\ -b & \text{if } b < 0 \end{cases}$$



In this region, $x < -5$

Thus: $x+5 < 0$, $x+2 < 0$

$\Rightarrow |x+5| = -(x+5)$

$|x+2| = -(x+2)$

So $|x+2| + |x+5| = 9$

$-(x+2) - (x+5) = 9$

$-2x - 7 = 9$

$-2x = 16$

$\boxed{x = -8}$

In this region, $-5 < x < -2$

Thus: $x+5 > 0$, $x+2 < 0$

$\Rightarrow |x+5| = x+5$

$|x+2| = -(x+2)$

So

$|x+5| + |x+2| = 9$

$x+5 - (x+2) = 9$

$3 \neq 9$

no solution

In this region, $x > -2$

Thus: $x+5 > 0$, $x+2 > 0$

$\Rightarrow |x+5| = x+5$

$|x+2| = x+2$

So

$|x+5| + |x+2| = 9$

$x+5 + x+2 = 9$

$2x = 2$

$\boxed{x = 1}$

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Question 6.

a) (5 points) What is the equation of the line that passes through $(-2, 1)$ and $(4, -3)$?

Slope is $m = \frac{-3-1}{4-(-2)} = \frac{-4}{6} = -\frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{2}{3}(x - (-2))$

$y = -\frac{2}{3}x - \frac{4}{3} + 1$

$\boxed{y = -\frac{2}{3}x - \frac{1}{3}}$

b) (5 points) What is the equation of the line *perpendicular* to the line given by

$$y = \frac{1}{4}x + 1$$

and which crosses the y-axis at height -7? ← y-intercept

Slope perpendicular to a slope of $\frac{1}{4}$ is $m = -4$.

$\boxed{y = -4x - 7}$.

Question 7.

a) (6 points) What is the vertex of the parabola defined by $y = x^2 - 8x + 14$?

highest or lowest point on the parabola
(~~...~~ depending on if parabola faces up or down)

Complete the square: $y = x^2 - 8x + 16 - 16 + 14$

$$y = (x-4)^2 - 16 + 14$$

$$y = \underbrace{(x-4)^2}_{\text{always } \geq 0 \text{ because squared}} - 2$$

Thus the smallest the y-value can be is -2, and this happens when $x = 4$.

$$\boxed{(4,-2)}$$

b) (6 points) At what point(s), if any, does the above parabola cross the x-axis (i.e. what are the roots of the equation)?

Crossing x-axis $\Rightarrow y = 0$.

Must solve: $0 = x^2 - 8x + 14$

Can either use quadratic formula, or completing the square. From completing the square above, we see

$$0 = (x-4)^2 - 2$$

$$2 = (x-4)^2$$

$$\pm\sqrt{2} = x - 4$$

$$\boxed{4 \pm \sqrt{2} = x}$$

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Question 8.

a) (3 points) What is the equation of the circle defined as the set of points (x, y) in the plane that are distance 3 from the point $(-2, 8)$?

$$(x+2)^2+(y-8)^2=3^2$$

$$\boxed{(x+2)^2+(y-8)^2=9}$$

b) (1 point) What is the circumference of this circle?

$C=2\pi r$, the radius $r=3$

$$\boxed{C=6\pi}$$

c) (1 point) What is the area of this circle?

$$A=\pi r^2$$

$$\boxed{A=9\pi}$$

d) (2 points) If I stretched this circle by a factor of $\frac{1}{3}$ in the horizontal direction and a factor of 2 in the vertical direction, what would the area of this new shape be?

New area = (horizontal stretch) × (vertical stretch) × (old area)

$$= \frac{1}{3}\times 2\times 9\pi$$

$$= \boxed{6\pi}$$

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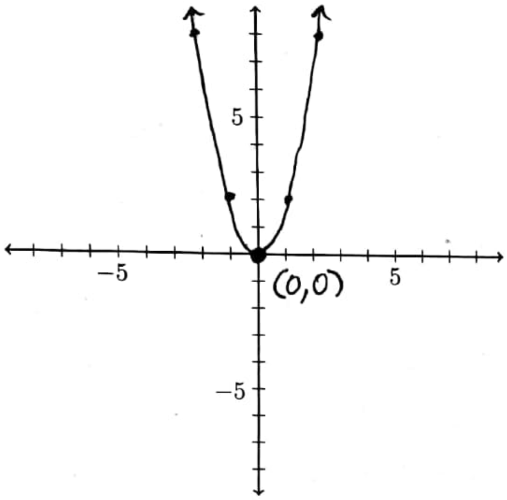
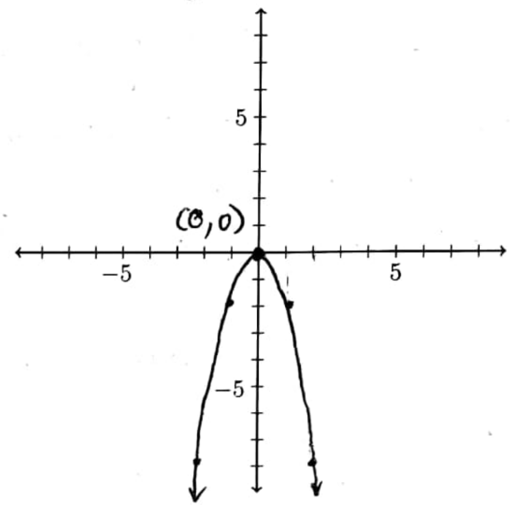
Question 9. (3 points per graph; 12 points total) Sketch the following. *Your sketch does not need to be precise, but it should capture the overall behavior of the graph.* Label the coordinates of the vertex on each graph.

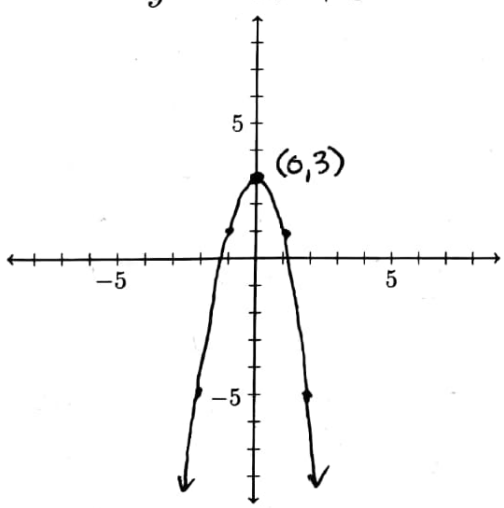
$$y = 2x^2$$



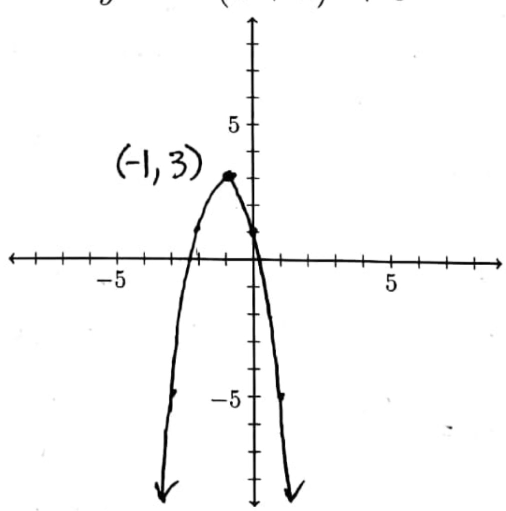
$$y = -2x^2$$



$$y = -2x^2 + 3$$



$$y = -2(x+1)^2 + 3$$



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