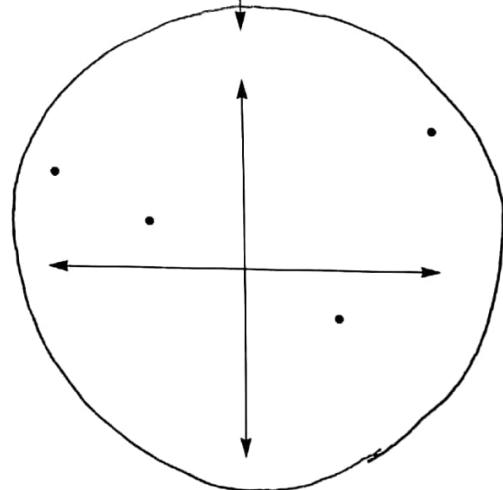
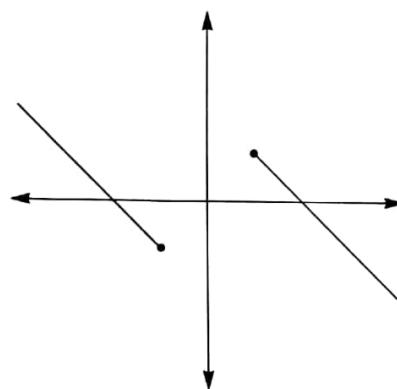
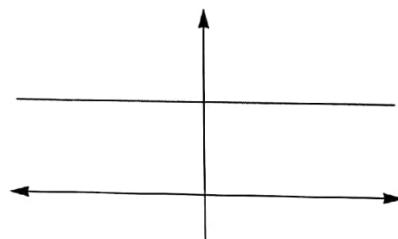
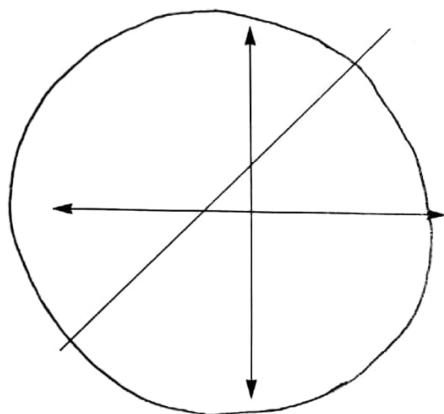
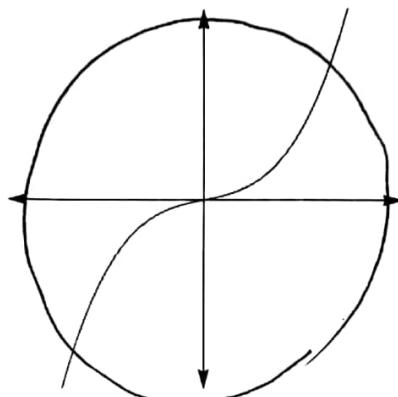
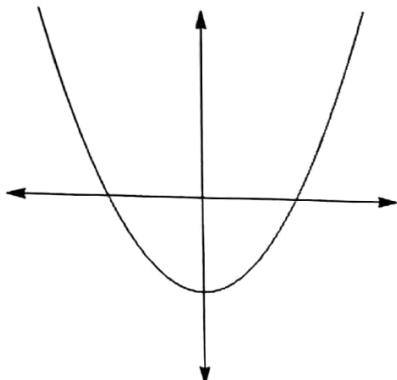
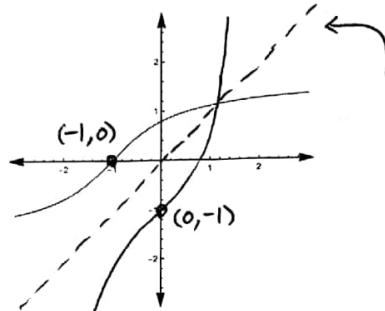


Question 1. (6 points) Which of the following functions are one-to-one? Circle all that apply.
one-to-one \Leftrightarrow passes horizontal line test

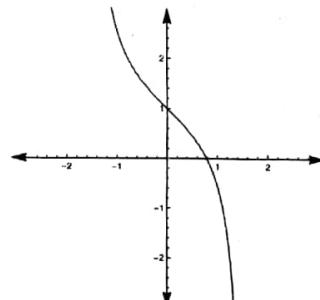
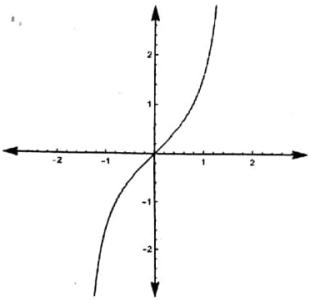
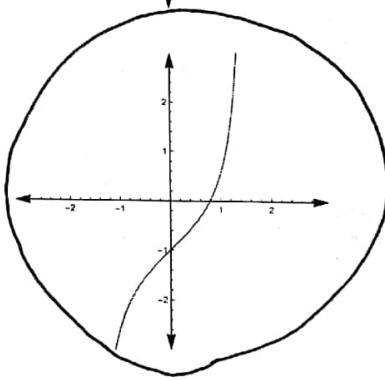
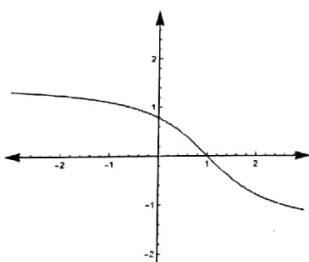
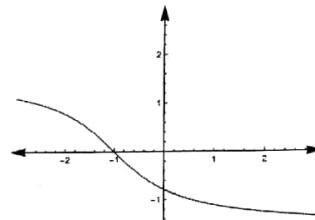
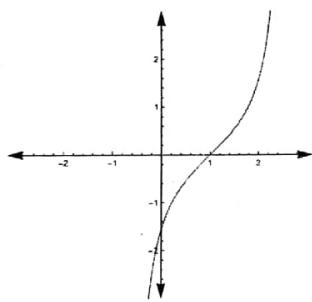


Question 2. (4 points) The graph of a function $h(x)$ is given by:



inverse is reflection
about the line $y=x$.

Which of the following could be the graph of $h^{-1}(x)$? Circle one.



Question 3. (1 point each; 15 points total) For each statement, circle "T" for True or "F" for False. You do not need to justify your answers.

T / F : $0^0 = 0$ }
 T / F : $0^0 = 1$ } 0^0 is undefined

(T) / F : $(-27)^{2/3} = 9$ $((-27)^{1/3})^2 = (-3)^2 = 9$

T / F : $(-4)^{3/2} = -8$ $((-4)^{1/2})^3$ → undefined, because there is no real # x such that $x^2 = -4$

T / F : If g is one-to-one, then $g^{-1}(x) = \frac{1}{g(x)}$. inverse function is not the same as reciprocal

T / F : For any positive x and y , $\ln(x+y) = \ln(x) + \ln(y)$.

(T) / F : For any positive x and y , $\ln(xy) = \ln(x) + \ln(y)$.

T / F : For any positive x and y , $\ln(x+y) = \ln(x) \ln(y)$.

} only the second of these is a log rule. The others can be shown false by counterexample

T / F : The polynomial $p(x) = x^3 + 7x^2 + x + 1$ has four zeros. # of zeros \leq degree

(T) / F : The polynomial $q(x) = x^5 + 6x^4 - 3x^3 - 3x^2 + x - 2$ has a factor of $(x - 1)$.

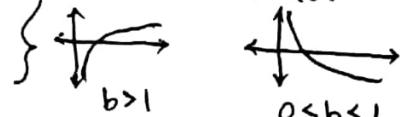
$(x-a)$ is a factor $\Leftrightarrow a$ is a zero. Note: $q(1) = 0$.

T / F : The function $h(x) = 2^{2x} + 3 \cdot 2^x + 1$ is a polynomial.

} polynomials cannot contain exponential functions.

T / F : For any $b > 0$, $b \neq 1$, the function $f(x) = \log_b x$ is increasing.

(T) / F : For any $b > 1$, the function $f(x) = \log_b x$ is increasing.

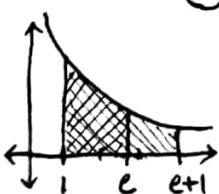


(T) / F : If $r(x) = \frac{x^8 - 12x^2 + 1}{3x + 4}$ is written as $r(x) = G(x) + \frac{R(x)}{3x + 4}$, where G and R are polynomials and $\deg(R) < 1$, then the degree of G is 7.

} use polynomial long division
(only have to do one step)

(T) / F : The area under the graph of $f(x) = \frac{1}{x}$ and above the x -axis between $x = 1$ and $x = e + 1$ is greater than 1.

The number e is defined such that the area under $\frac{1}{x}$ and above the x -axis between 1 and e is exactly 1.
There is even more area between 1 and $e+1$.



Question 4. Let f and g be defined by the tables below.

x	$f(x)$
1	2
2	4
3	3

x	$g(x)$
2	3
3	2
4	1

Compute the following:

a) (1 point) $f^{-1}(2)$

$$f(1) = 2 \Leftrightarrow f^{-1}(2) = \boxed{1}$$

b) (1 point) $(g \circ f)(3)$

$$g(f(3)) = g(3) = \boxed{2}$$

c) (1 point) $(g \circ g^{-1})(2)$

$$(g \circ g^{-1})(x) = x \text{ for any } x, \text{ so } (g \circ g^{-1})(2) = \boxed{2}$$

d) (2 points) $(g^{-1} \circ f^{-1} \circ f)(1)$

$$g^{-1}(f^{-1} \circ f)(1) = g^{-1}(1) = \boxed{4}$$

because $g(4) = 1$

e) (3 points) $(f \circ g)^{-1}(4)$

x	$(f \circ g)(x)$
2	3
3	4
4	2

Ex: because
 $f(g(2)) = f(3) = 3$
 same reasoning applies to all

$$(f \circ g)(3) = 4$$



$$(f \circ g)^{-1}(4) = \boxed{3}$$

Note: $(f \circ g)^{-1} \neq f^{-1} \circ g^{-1}$!

Question 5. For parts (a) through (i), let

$$f(x) = 3x^2 + 2 \quad \text{and} \quad g(x) = \frac{2-x}{x+5}.$$

Assume the domain for each function is the largest set of real numbers for which the formula is defined and produces a real number.

a) (2 points) Is f one-to-one? Why or why not? Possible correct answers include:

No, because the graph of f is a parabola, which does not pass the horizontal line test.

No, because there are multiple inputs for a single output.
Ex: $f(1) = 5 = f(-1)$.

b) (5 points) The function g is one-to-one. Find a formula for g^{-1} .

$$\text{Solve for } x: \quad y = \frac{2-x}{x+5}$$

$$(x+5)y = 2-x$$

$$xy + 5y = 2 - x$$

$$xy + x = 2 - 5y$$

$$x(y+1) = 2 - 5y$$

$$x = \frac{2-5y}{y+1}$$

$$\therefore \boxed{g^{-1}(y) = \frac{2-5y}{y+1}} \quad (\text{or } g^{-1}(x) = \frac{2-5x}{x+1})$$

variable used does not matter, but cannot use both x and y to define the function!

c) (3 points) What is the domain of g ? The denominator of g can't be zero, so $x+5 \neq 0 \Rightarrow x \neq -5$. Correct ways of writing this include: "all real numbers except -5 ", $(-\infty, -5) \cup (-5, \infty)$, or $\{x | x \neq -5\}$.

must use ")" and not "]" because -5 is not included in the domain.

d) (3 points) What is the domain of g^{-1} ?

Same reasoning as above. Correct ways of writing the answer include: $(-\infty, -1) \cup (-1, \infty)$, $\{y | y \neq -1\}$, $\{x | x \neq -1\}$, or "all real numbers except -1 ".

e) (2 points) What is the range of g ?

$$\text{range of } g = \text{domain of } g^{-1} : (-\infty, -1) \cup (-1, \infty)$$

* or any other correct way of writing the domain/range as described in parts (c) and (d).

f) (2 points) What is the range of g^{-1} ?

$$\text{range of } g^{-1} = \text{domain of } g : (-\infty, -5) \cup (-5, \infty)$$

g) (5 points) Find a formula for $f \circ g$. Simplify as much as possible.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{2-x}{x+5}\right)$$

$$= 3\left(\frac{2-x}{x+5}\right)^2 + 2$$

$$= 3 \frac{(2-x)^2}{(x+5)^2} + 2$$

$$= 3 \cdot \frac{(4-4x+x^2)}{(x^2+10x+25)} + 2$$

$$= \frac{3x^2-12x+12}{x^2+10x+25} + \frac{2x^2+20x+50}{x^2+10x+25}$$

Note: $(a+b)^2 \neq a^2+b^2$!
} a lot of people made
this mistake

$$(f \circ g)(x) = \frac{5x^2+8x+62}{x^2+10x+25}$$

h) (5 points) Find a formula for $g \circ f$. Simplify as much as possible.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) = g(3x^2 + 2) \\&= \frac{2 - (3x^2 + 2)}{3x^2 + 2 + 5} \quad \leftarrow -(3x^2 + 2) = -3x^2 - 2 \\&= \frac{2 - 3x^2 - 2}{3x^2 + 7}\end{aligned}$$

$$(g \circ f)(x) = \frac{3x^2}{3x^2 + 7}$$

Note: cannot cancel $3x^2$ from top and bottom! These are terms in a sum, not factors in a product.

So $\frac{a \cdot b}{a \cdot c} = \frac{b}{c}$ ✓, but $\frac{a+b}{a+c} \neq \frac{b}{c}$!

i) (5 points) Find all asymptotes of g .

Vertical asymptotes occur where the denominator is zero. In part (c) we found that this occurs at $x = -5$.

vertical asymptote: $x = -5$

Since the degree of the numerator = degree of denominator (both are 1), the horizontal asymptote occurs at the ratio of the coefficients of the highest powers of x :

For large x , $\frac{2-x}{x+5} \approx \frac{-x}{x} = -1$.

so horizontal asymptote: $y = -1$

Question 6. Simplify the following expressions as much as possible.

a) (5 points) $\left(\frac{x^3y^{-6}}{(xy^2)^2}\right)^{-1}$

$$= \left(\frac{x^3y^{-6}}{x^2y^4}\right)^{-1}$$

$$= (x^{3-2}y^{-6-4})^{-1}$$

$$= (xy^{-10})^{-1}$$

$$= \boxed{x^{-1}y^{10}} \text{ or } \boxed{\frac{y^{10}}{x}}$$

* there are multiple correct ways to arrive at the same answer in each of these problems.

(either form accepted)

b) (5 points) $\log_2(2^x) - \log_2\left(\frac{1}{8}\right)$

$$= x\log_2 2 - \log_2(2^{-3})$$

$$= x - (-3)$$

$$= \boxed{x+3}$$

* I've shown 2 ways of getting the answer here, but there are more!

$$\log_2(2^x) - \log_2\left(\frac{1}{8}\right)$$

$$= \log_2\left(\frac{2^x}{\sqrt[3]{8}}\right)$$

$$= \log_2(8 \cdot 2^x)$$

$$= \log_2(2^3 \cdot 2^x)$$

$$= \log_2(2^{x+3})$$

$$= x+3$$

c) (5 points) $\ln\left(\frac{1}{2}\ln(e^3) - \ln(\sqrt{e})\right)$

$$= \ln\left(\frac{3}{2}\ln(e) - \ln(e^{1/2})\right)$$

$$= \ln\left(\frac{3}{2} - \frac{1}{2}\ln(e)\right)$$

$$= \ln\left(\frac{3}{2} - \frac{1}{2}\right)$$

$$= \ln(1)$$

$$= \boxed{0}$$

* as before, there are different methods to get the same result.