
HOMEWORK ASSIGNMENT

During the course, the HW assignment may be slightly changed. Please, watch the current assignment.

In problems marked by *, the student may skip final calculations, stopping at the moment when the calculations remained are obvious.

1. 4.8: 2, 3, 9, 12, 16 (Hint: Write 2 as 1 + 1 and look at what happened), 17, 19, 22, 28, 29*, 39, 43*, 46* (optional), 56*.
   5.1: 1a, 4*, 16*, 22.
   5.2: 31b*, 33*, 34*, 38*, 39 (Hint: The answer should be immediate);
   A1. Say what \( \int_{-\pi/2}^{\pi/2} \sin^3 x \, dx \) equals to. (Hint: If you started to calculate something, then you are on a wrong way);
   5.3: 1, 4*, 6*, 7*, 13, 15, 16*, 19*, 31, 49 (just write the integral you are going to compute), 50*, 53, 54.
   5.4: 2cd, 7, 11, 12, 13, 14, 19abc, 21;
   A2. Show that the function \( F(x) = \int_0^x e^{t^2} \, dt \) is convex (concave upward) for \( x > 0 \). (Hint: You do not have to compute the second derivative precisely – just figure out (looking at the first derivative) whether the second derivative is positive or negative.)
   A3. Give an expression for a function \( F(x) \) such that \( F'(x) = \cos^3 x \), and \( F(0) = 2 \). (Hint: The word “expression” does not mean a precise representation in terms of elementary functions.)

Due to July 10.

2. 5.5: 3, 5, 7*, 9, 12*, 15, 16, 21, 31*, 32, 33* (Hint: Use the “doubled angle” formula), 34, 41, 47, 51*, 55.
   A4. Compute \( \int \frac{\ln^2 x}{x} \, dx \), \( \int \frac{1}{x \ln^2 x} \, dx \), \( \int e^x \frac{1}{x \ln^2 x} \, dx \).
   5.6: 2, 3, 5, 6, 7 (Hint: You may differentiate by parts twice), 13, 17, 20, 25.
   5.7: 1, 2, 11, 16 (Advice: Do the substitution \( x = 4 \sin t \), and think about a proper range for \( t \)), 21, 23, 25, 29 (Advice: You may rather than following the advice in the book, just present the numerator as \( x - 6 + 6 \) and after that simplify the integrand);
   A5. Compute the following integrals skipping, if you wish, final calculations:
      (a) \( \int_0^{\pi/2} \sin^5 x \cos^3 x \, dx \).
\begin{align*}
\text{(b)} & \quad \int_0^{\pi/2} x \cos x \, dx. \\
\text{(c)} & \quad \int_3^4 \frac{3x + 2}{(x - 2)(x + 2)} \, dx.
\end{align*}

A6. Compute the integrals:

\begin{align*}
\int_0^3 \sqrt{9 - x^2} \, dx, \quad & \quad \int_0^3 x \sqrt{9 - x^2} \, dx, \quad \int_0^{3/2} \sqrt{9 - 4x^2} \, dx, \quad \int_0^{3/2} x \sqrt{9 - 4x^2} \, dx.
\end{align*}

For the last integral, you do not have to provide final calculations. (\textit{Hint:} These integrals look similar but, as a matter of fact, they require different methods to apply.)

Due to July 17.

3. 5.10 (In problems marked by the asterisk \(*\), computing the integral is optional): 1, 2ab, 5, 6*, 10 (look at the lower limit), 11, 13, 15, 17*, 19, 21*, 22*, 23, 24*, 31 (consider \( \int_0^1 \)), think which substitution is more convenient: \( u = e^x \) or \( u = e^x - 1 \), 43, 46, 48, 50, 51 (Pay attention to this problem).

A7. For integrals below, figure out whether they converge or diverge. In the former case, calculate these integrals, and in the latter case, justify your answer. You do not have to provide final answers: just make sure that you can solve these problems.

\begin{align*}
\text{(a)} & \quad \int_1^\infty \frac{1}{x^{1.01}} \, dx. \\
\text{(b)} & \quad \int_1^\infty \frac{1}{x^{0.99}} \, dx. \\
\text{(c)} & \quad \int_e^\infty \frac{1}{x \sqrt{\ln x}} \, dx. \\
\text{(d)} & \quad \int_e^1 \frac{1}{x (\ln x)^{10}} \, dx. \\
\text{(e)} & \quad \int_0^1 \frac{1}{x^{1.01}} \, dx. \\
\text{(f)} & \quad \int_0^1 \frac{1}{x^{0.99}} \, dx. \\
\text{(g)} & \quad \int_0^1 \frac{1}{(1 - x)^{1/2}} \, dx. \text{ (\textit{Advice:} Do the substitution } w = 1 - x.)
\end{align*}

A8. Not computing the integrals below, just say whether they converge or diverge.

\begin{align*}
\text{(a)} & \quad \int_5^\infty \frac{x^4 + x^2 + 7}{x^8 + x^7 + 5} \, dx. \\
\text{(b)} & \quad \int_3^\infty \frac{x^4 + 30x + 1}{x^5 + x + 1} \, dx. \\
\text{(c)} & \quad \int_0^1 \frac{1 + x^3}{x + x^4} \, dx.
\end{align*}
(d) \[ \int_{0}^{1} \frac{1 + x^3}{\sqrt{x} + x^4} \, dx. \]

(e) \[ \int_{0}^{1} \frac{1}{(1 - x)^{1/2} + (1 - x)^2} \, dx. \]

Due to July 24.

4. **6.1**: 1, 2, 5, 9, 10°, 16°, 23 (optional).

**6.2**: 2, 3, 10°, 33, 45 (b).

5. **7.1**: 1, 2, 3, 5, 7, 10.

**7.3** (In problems marked by the asterisk * you may (though do not have to) stop when you explicitly see how you will finish the problem): 1, 3, 5 (*Hint*: A solution may be implicit). 10, 11°, 12°, 16°, 17° (*Hint*: A bit challenging; multiply both sides by \( \cos x \) and move all terms involving \( y \) to the l.-h.s.), 18.

Due to.

**7.4**: 3ab, 5bc (optional).

6. **8.1** (If you can solve a problem in mind, then you may skip the proof): 5, 11, 12, 13, 15, 19, 28, 31, 33.

**8.2** (If you can solve a problem in mind, then you may skip the proof): 10, 14, 15, 18, 35ab, 41, 43, 53 (the problem is a bit challenging; multiply both sides of the equation by \((1 + c)^2\) and realize which sum you will have on the left after that).

**8.8** (You should find approximating polynomials manually, while in graphing you are recommended to use software (say, a graphic calculator)): 1, 2, 4, 5, 7, 19;

A7. Write a Taylor approximation of order \( n \) for \( \ln x \) at \( a = 1 \). (*Hint*: To avoid superfluous calculation, write first \( \ln x = \ln(1 + x - 1) \), and then use the approximation for \( \ln(1 + x) \).)

A8. Write a Taylor approximation of order \( n \) for \( \frac{\sin x}{x} \) at \( a = 0 \). If it seems difficult for you to consider an arbitrary \( n \), set \( n = 4 \). (*Hint*: You may avoid superfluous calculations.)

A9. Write a Taylor approximation of order \( n \) for \( \frac{1 - \cos x}{x} \) at \( a = 0 \). If it seems difficult for you to consider an arbitrary \( n \), set \( n = 3 \). (*Hint*: You may avoid superfluous calculations.)

Due to.