Math 10B, July-2021.
HOMEWORK ASSIGNMENT

During the course, the HW assignment may be slightly changed. ***Please, watch the current assignment***.

Excepting the first item, all problems are from Volume 2. In problems marked by *, the student may skip final calculations, stopping at the moment when the calculations remained are obvious.

1. **4.10, Volume 1**: 465, 468-472, 475, 476, 479, 484, 490, 491, 493, 495, 498, 499, 501.

   A1. Let a function
   $$f(x) = \begin{cases} -2 & \text{if } 0 \le x < 2 \\ 1 & \text{if } 2 \le x \le 3. \end{cases}$$
   Graph this function. After that, graph its antiderivative $F(x)$ such that $F(0) = -1$. (***Hint***: We solved a similar problem in lectures.)

   1.1: 1, 3, 6, 12, 13,

   1.2: 76-80, 82, 88, 90, 93. 98.

   1.3: 170*, 177*, 179, 182, 183, 185, 148, 150, 153, 156, 159, 161. (Such an order is chosen on purpose),

   A2. Show that the function $F(x) = \int_0^{x^2} e^{t^2} dt$ is convex (concave upward) for $x > 0$. (***Hint***: You do not have to compute the second derivative precisely – just figure out (looking at the first derivative) whether the second derivative is positive or negative.)

   A3. Give *an expression* for a function $F(x)$ such that $F'(x) = \cos x^3$, and $F(0) = 2$. (***Hint***: The word "expression" does not mean a precise representation in terms of elementary functions.)

   A4. Say what $\int_{-\pi/2}^{\pi/2} \sin x^3 dx$ equals to. (***Hint***: If you started to calculate something, then you are on a wrong way.)

   A5. Compute in mind $\int_{-1}^{1} x^4 dx$ taking into account that the integrand is an even function. Do the same for $\int_{-2}^{2} x^4 dx$.

   Due to July 9.

2. **1.4**: 214, 226*.

   1.5: 256, 260-262, 265, 271-272, 276-278, 261, 292, 294. 297.

   1.6: 320, 321, 322, 324, 328, 336, 337, 348.

A6. Compute $\int \frac{\ln^2 x}{x} dx$, $\int \frac{1}{x \ln^2 x} dx$, $\int_e^{e^2} \frac{1}{x \ln^2 x} dx$.

1.7: 391, 397-400.

2.1: 1, 2, 4, 6, 14.

2.2: 75, 80, 86, 99, 101.

Due to July 16.

3. **2.8**:

A7. You invested \$10,000 in an account with an annual interest rate of 6% , and the interested is compounded continuously. If you withdraw no money from the account, at which time will your capital exceed \$30,000 ?

3.1: 6-9, 12-16, 20, 23 ($u = \ln x, x = e^u$), 25-27, 41-41.

A8. Compute the integral $\int_0^{\pi/2} x \cos x \, dx$.

3.2: 71-74, 76, 7981, 85.

A9. Compute the integral $\int_0^{\pi/2} \sin^5 x \ \cos^3 x \, dx$.

3.3: 134, 136

A10. Compute the integrals

$$\int_0^3 \sqrt{9 - x^2} \, dx, \ \int_0^3 x \sqrt{9 - x^2} \, dx, \ \int_0^{3/2} \sqrt{9 - 4x^2} \, dx, \ \int_0^{3/2} x \sqrt{9 - 4x^2} \, dx.$$

For the last integral, you do not have to provide final calculations. (*Hint*: These integrals look similar but, as a matter of fact, they require different methods to apply.)

3.4: 182-183, 187, 188, 202-204 (*Hint*: Factor the denominators.)

A11. Compute the integral

$$\int_3^4 \frac{3x + 2}{(x - 2)(x + 2)} dx.$$

For the last integral, you do not have to provide final calculations.

Due to July 23.

4. **3.7** (In problems marked by the asterisk *, computing the integral is optional): 347*, 348*, 350, 351*, 255,*, 356, 357, 359. 360. 365, 371-373, 375, 378.

   A12. For integrals below, figure out whether they converge or diverge. In the former case, calculate these integrals, and in the latter case, justify your answer. You do not have to provide final answers: just make sure that you can solve these problems.

   (a) $\displaystyle\int_1^\infty \frac{1}{x^{1.01}}dx.$

   (b) $\displaystyle\int_1^\infty \frac{1}{x^{0.99}}dx.$

   (c) $\displaystyle\int_e^\infty \frac{1}{x\sqrt{\ln x}}dx.$

   (d) $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^{10}}dx.$

   (e) $\displaystyle\int_0^1 \frac{1}{x^{1.01}}dx.$

   (f) $\displaystyle\int_0^1 \frac{1}{x^{0.99}}dx.$

   (g) $\displaystyle\int_0^1 \frac{1}{(1-x)^{1/2}}dx.$ (*Advice*: Do the substitution $w = 1 - x$.)

   A13. Not computing the integrals below, just say whether they converge or diverge.

   (a) $\displaystyle\int_5^\infty \frac{x^4+x^2+7}{x^8+x^7+5}dx.$

   (b) $\displaystyle\int_3^\infty \frac{x^4+30x+1}{x^5+x+1}dx.$

   (c) $\displaystyle\int_0^1 \frac{1+x^3}{x+x^4}dx.$

   (d) $\displaystyle\int_0^1 \frac{1+x^3}{\sqrt{x}+x^4}dx.$

   (e) $\displaystyle\int_0^1 \frac{1}{(1-x)^{1/2}+(1-x)^2}dx.$

   Due to July .

5. **4.1**: 1-6, 8-10, 12-28, 31, 37, 432 42. 16.

   4.2: 66, 67, 70,

   4.3: 119, 120, 123, 124, 133, 134.

   5.1: 2, 7, 8, 23, 24, 27, 28.

   A14. Find limits below.

   (a) $\lim_{n\to\infty} \frac{n}{n+1}$

(b) $\lim_{n\to\infty} \frac{n^2}{n^2+7}$

(c) $\lim_{n\to\infty} \frac{2n^2}{3n^2+7}$

(d) $\lim_{n\to\infty} e^{\frac{n}{n+1}}$

(e) $\lim_{n\to\infty} e^{-\frac{n}{n+1}}$

5.2: 67, 68, 81, 84, 89, 90, 93, 94, 96.

A15. For which x's does the series

$$1 + \frac{x}{5} + \frac{x^2}{25} + \frac{x^3}{125} + \cdots \frac{x^n}{5^n} + \cdots$$

converge? To which function of x does it converge?

Due to July .