Math 193a,  Summer 2019.  HOMEWORK ASSIGNMENT

During the course, the HW assignment may be slightly changed. Please, watch the current assignment.

When solving problems marked by *, you do not have to provide final calculations: it is enough to know how to solve these problems. Certainly, you may nevertheless do calculations (for example, in order to compare your answer with that in the book), and in any case, you should make sure that you will be able to do it if needed.

1. Chapter 1: 1, 3abd (there is a typo in 3b: please, replace \(x^2\) by \(X^2\)), 3c (optional), 12, 13, 18, 19* (it is enough to write an equation for the premium), 22, 23, 24 (Hint: A concave function does not have to be differentiable), 25 (The answer is “yes”, look up Exercise 24), 26 (Hint: Which property of the utility functions should you check?), 27 (Advice: Take also a look at Proposition 2 on p.95), 30a, 30bc (We skipped risk averse measures (Section 3.4.3), so just try to give heuristic answers without rigorous justification (for example, thinking which function from the given two ones is “more concave”. However, if you wish, you can read Section 3.4.3 and provide rigorous answers.), 31 (optional), 38 (Advice: Consider \(E[f(X_1)] - E[f(X_2)]\), where \(X_1\) and \(X_2\) are the r.v.’s defined, and \(u(x)\) is a utility function. After some simplification, use the fact that \(u(x)\) is concave.), 39 (realize that this is a generalization of Problem 38; a precise solution is optional), 61 (Hint: In the first case, \(F_0(x) = 1 - e^{-x/m}\), and in the second case, \(F_0(x) = \frac{x^2}{2m}\) for \(x \in [0, 2m]\) and \(F_0(x) = 1\) for \(x \geq 2m\). So, in both cases, the integration in (5.1.4) is easy.).

Due to July 12.

2. Chapter 2: 1 (absolutely mandatory), 2a (Hint: Distinguish the cases where the multiplier is positive and negative), 2bc, 2d (Problems (ii) and (iii) are optional), 5a (Hint: The problem is easy: look at Exercise 1), 7, 9ab (in 9b, you may restrict yourself to \(E\{Y\}\), 10* (You do not have to do final calculations and may just write all formulas with particular data plugged in), 11a (optional), 11b, 11c* (Hint: The formulas you need are prepared in p.153), 11d, 18, 19a, 19b (optional), 20a, 21 (optional), 22 (Hints: In Problem (b), the answer is immediate; in Problem (c), you should explain due to which property of the Gamma-distribution the two probabilities mentioned are not the same), 23, 24a, 26, 27, 28-29 (In addition, in both problems, proceeding from the Central Limit Theorem, argue that the random variables under consideration are asymptotically normal for large \(n\)), 30, 33, 34, 35, 36, 47ab, 48abcdf.

Additional problems:

(a) Let a r.v. \(X_1\) assume values 7 and 11 with probabilities 0.3 and 0.7, respectively, and a r.v. \(X_2\) assume values 7 and 11 with probabilities 0.7 and 0.3, respectively. Write the m.g.f.’s of these r.v.’s. Write the m.g.f. of \(X_1 + X_2\) in the case where \(X_1\) and \(X_2\) are independent.

(b) What is the distribution of the sum of 30 independent exponential r.v.’s with the same mean value? Write the probability density function for the case when the mean of the exponential r.v.’s above equals 3. (Hint: The number 3 given is a mean rather than the parameter \(a\) of the exponential distribution.)

Due to July 19.

3. Chapter 3: 2, 3, 4, 6a (recall the Poisson theorem, find just the lambda, approximating first the numbers of claims in each separate group by Poisson variables (finding, in particular, the corresponding \(\lambda\)’s for each group), and then adding up these Poisson variables to get the total number of claims), 6b (the numerical answer will be needed for the next question – you may use Excel for calculations (the command is =POISSON(.); for details click \(f_x\), type “Poisson”, and—if needed–read explanations)), 6c (just look at the
answer at the end of the book: it is based on your calculations for Exercise 6b), 12, 14, 21, 22a, 27a (there is a typo in the statement of this problem: the symbols $K_1, K_2, K_3$ should be replaced by $N_1, N_2, N_3$; 

Hint: In accordance with the theory we have built, the r.v.'s $N_i$'s are independent), 28, 29, 32–36 (In all these exercises, you do not have to write detailed solutions. You may skip calculations if you are sure that your solution is correct, and you are even encouraged to write something like “the same as in the previous problem with the exception that . . . ”).

Regarding Poisson and $\Gamma$-distributions, recollect Problems 28-29 from Chapter 2 and proceeding from the Central Limit Theorem, argue that in both problems the random variables under consideration are asymptotically normal for large $n$).

Regarding Exercise 27 and others (as well as the whole material of Section 3.1,2), it makes sense to repeat that the values $x_i$ in (3.1.13) do not impact on the distributions of the r.v.'s $N_i$. As a matter of fact, we consider $l$ types of claims (whatever these types are), and $N_i$ is the number of claims (or “arrivals”) of the type $i$. Proposition 10 on p. 220 states that the r.v.'s $N_i$ are independent, and have Poisson distributions with $\lambda_i = \lambda p_i$.

An additional problem: The number of accidents for a risk portfolio and a certain period of time has the Poisson distribution with parameter $\lambda_1$. Each accident may cause several claims. Let the numbers of claims corresponding to particular accidents are independent Poisson r.v.'s with parameter $\lambda_2$. Let $N$ be the total number of claims. (In other words, $N$ is the sum of Poisson r.v.'s and the number of these r.v.'s (that is, the terms in the sum) is also Poisson.) Such a distribution is called Poisson-Poisson. Show that this is a particular case of the compound Poisson distribution. Find $E\{N\}$ and $\text{Var}\{N\}$. (Hint: You should use the same scheme from Section 1, Chapter 3. Realize also what is $E\{X^2\}$ if $X$ is a Poisson r.v. with a parameter of $\lambda$).

Due to July 26.

4. **Chapter 4:** 1a (do not write long explanations; just understand for yourself), 2 (do not write long explanations; just understand for yourself), 9, 10°, 11, 12 (questions on correlations are optional, but if you know what it is (hopefully, you do), then it is recommended to consider these questions also), 13, 15, 16ab, 16c*, 16d (Advice: Recall that for large $\lambda$'s, the Poisson distribution is well approximated by a normal distribution), 17 (give just an heuristic explanation; compare it with the case of a homogeneous Poisson process), 18, 19*, 20 (it is enough to write an inequality), 27°, 29.

Due to July 29.

5. **Chapter 6:** 2, 4 (only Exercises 30, 33-36 from Chapter 2), 5a*, 5c, 6a (Hint: At the end of your reasoning, you should look at formula (2.1.10)), 7a (Hint: Look at Fig. 3b, p. 344, and observe that the larger $c$, the steeper the slope of the curve at the origin.), 8 (Answer: It suffices to do it just one time. Advice: Think about the substitute $y = z/a$), 10, 11°, 13ab (Write a correct equation for $\gamma$. Check just by substitution that $\gamma \approx 0.053$. Certainly, you may also solve the equation graphically or using software.), 14 (you may take a look at the answer at the end of the book.), 15.

Due to August 3.