Math 193a, HOMEWORK ASSIGNMENT

During the course, the HW assignment may be slightly changed. Please, watch the current assignment.

When solving problems marked by *, you do not have to provide final calculations: it is enough to know how to solve these problems. Certainly, you may nevertheless do calculations (for example, in order to compare your answer with that in the book), and in any case, you should make sure that you will be able to do it if needed.

1. ***Chapter 1***: 1, 3abd (there is a typo in 3b: please, replace ξ_2 by X_2), 3c (optional), 12, 13, 18, 19* (it is enough to write an equation for the premium), 22, 23, 24 (*Hint*: A concave function does not have to be differentiable), 25 (The answer is "yes", look up Exercise 24), 26 (*Hint*: Which property of the utility functions should you check?), 27 (*Advice*: Take also a look at Proposition 2 on p.95), 30a, 30bc (We skipped risk averse measures (Section 3.4.3), so just try to give heuristic answers without rigorous justification (for example, thinking which function from the given two ones is "more concave". However, if you wish, you can read Section 3.4.3 and provide rigorous answers.), 31 (optional), 38 (*Advice*: Consider $E\{u(X_1)\} - E\{u(X_2)\}$, where X_1 and X_2 are the r.v.'s defined, and $u(x)$ is a utility function. After some simplification, use the fact that $u(x)$ is concave.), 39 (realize that this is a generalization of Problem 38; *a precise solution is optional*), 61 (*Hint*: In the first case, $F_0(x) = 1 - e^{-x/m}$, and in the second case, $F_0(x) = \frac{x}{2m}$ for $x \in [0, 2m]$ and $F_0(x) = 1$ for $x \geq 2m$. So, in both cases, the integration in (5.1.4) is easy.).

   Due to July 9

2. ***Chapter 2***: 1 (absolutely mandatory), 2a (*Hint*: Distinguish the cases where the multiplier is positive and negative), 2bc, 2d (*Problems (ii) and (iii) are optional*), 5a (*Hint*: The problem is easy: look at Exercise 1), 7, 9ab (in 9b, you may restrict yourself to $E\{Y\}$), 10* (You do not have to do final calculations and may just write all formulas with particular data plugged in), 11a (*optional*), 11b, 11c* (*Hint*: The formulas you need are prepared in p.153), 11d, 18, 19a, 19b (optional), 20a, 21 (*optional*), 22 (*Hints*: In Problem (b), the answer is immediate; in Problem (c), you should explain due to which property of the Gamma-distribution the two probabilities mentioned are not the same), 23, 24a, 26, 27, 28-29 (In addition, in both problems, proceeding from the Central Limit Theorem, argue that the random variables under consideration are asymptotically normal for large n), 30, 33, 34, 35, 36, 47ab, 48abcdf.

   Additional problems:

   (a) Let a r.v. X_1 assume values 7 and 11 with probabilities 0.3 and 0.7, respectively, and a r.v. X_2 assume values 7 and 11 with probabilities 0.7 and 0.3, respectively. Write the m.g.f.'s of these r.v.'s. Write the m.g.f. of $X_1 + X_2$ in the case where X_1 and X_2 are independent. Do the same for the r.v. $X_1 - 2X_2$.

   (b) What is the distribution of the sum of 30 independent exponential r.v.'s with the same mean value? Write the probability density function for the case when the mean of the exponential r.v.'s above equals 3. (*Hint*: The number 3 given is a mean rather than the parameter a of the exponential distribution.)

   (c) You got in line for a taxi in an airport. There are 3 people before you. The (random) times between taxi arrivals are independent exponential r.v.'s with a mean of two minutes. Write the probability density of your (random) waiting time. Do the same if the times between taxi arrivals have a Γ-distributions with $\nu = 2$ and the same mean as above. (Hint: the mean of the Γ-distribution is ν/a.

   Due to July 18

3. ***Chapter 3***: 2, 3, 4, 6a (recall the Poisson theorem, find just the lambda, approximating first the numbers of claims in each separate group by Poisson variables (finding, in particular, the corresponding λ's for each group), and then adding up these Poisson variables to get the total number of claims), 6b (the numerical answer will be needed for the next question – you may use Excel for calculations (the command is =POISSON(.); for details click f_x, type "Poisson", and–if needed–read explanations)), 6c (just look at the answer at the end of the book: it is based on your calculations for Exercise 6b), 12, 14, 21, 22a, 27a (there is a typo in the statement of this problem: the symbols K_1, K_2, K_3 should be replaced by N_1, N_2, N_3; *Hint*: In accordance with the theory we have built, the r.v.'s N_i's are independent), 28, 29, 32–36 (In all these exercises, you may skip detailed calculations if you are sure that your solution is correct, and you are even encouraged to write something like "the same as in the previous problem with the exception that ... ").

   Additional remarks.

   (a) Regarding Poisson and Γ-distributions, recollect Problems 28-29 from *Chapter 2* and proceeding from the Central Limit Theorem, argue that in both problems the random variables under consideration are asymptotically normal for large n)

   (b) Regarding Exercise 27 and others (as well as the whole material of Section 3.1.2), it makes sense to repeat that the values x_i in (3.1.13) do not impact on the distributions of the r.v.'s N_i . As a matter of fact, we consider l types of claims (whatever these types are), and N_i is the number of claims (or "arrivals") of the type i. Proposition 10 on p. 220 states that the r.v.'s N_i are independent, and have Poisson distributions with $\lambda_i = \lambda p_i$.

   Additional problems:

   (a) The number of accidents for a risk portfolio and a certain period of time has the Poisson distribution with parameter λ_1. Each accident may cause several claims. Let the numbers of claims corresponding to particular accidents are independent Poisson r.v.'s with parameter λ_2. Let N be the total number of claims. (In other words, N is the sum of Poisson r.v.'s and the number of these r.v.'s (that is, the terms in the sum) is also Poisson.) Such a distribution is called *Poisson-Poisson*. Show that this is a particular case of the compound Poisson distribution. Find $E\{N\}$ and $Var\{N\}$. (*Hint*: You should use the same scheme from Section 1, Chapter 3. Realize also what is $E\{X^2\}$ if X is a Poisson r.v. with a parameter of λ).

   (b) You have come to a service facility with two counters. (For example, you have come to a store to return a good that you had recently bought, and there are two counters for this.) Both counters are busy, but there is nobody in line. Service times are i.i.d. (independent identically distributed) exponential r.v.'s. You noticed that one of two customers who are already being served, is your friend David. What is the probability that you will leave the facility before (earlier than) David? *Hint*: The problem may be solved in mind and requires no "serious" calculations.

   Due to July 23.

4. ***Chapter 4***: 1a (do not write long explanations; just understand for yourself), 2 (do not write long explanations; just understand for yourself), 9, 10*, 11, 12 (questions on correlations are *optional*, but if you know what it is (hopefully, you do), then it is recommended to consider these questions also), 13, 15, 16ab, 16c*, 16d (*Advice*: Recall that for large λ's, the Poisson distribution is well approximated by a normal distribution), 17 (give just an heuristic explanation; compare it with the case of a homogeneous Poisson process), 18, 19* , 20 (it is enough to write an inequality), 27* , 29.

5. ***Chapter 6***: 2, 4 (only Exercises 30, 33-36 from Chapter 2), 5a* , 5c, 6a (*Hint*: At the end of your reasoning, you should look at formula (2.1.10)), 7a (*Hint*: Look at Fig. 3b, p. 344, and observe that the larger c, the steeper the slope of the curve at the origin.), 8 (*Answer*: It suffices to do it just one time. *Advice*: Think about the substitute $y = z/a$.), 10, 11* , 13ab (Write a correct equation for γ. Check just by substitution that $\gamma \approx 0.053$. Certainly, you may also solve the equation graphically or using software.), 14 (you may take a look at the answer at the end of the book.), 15.

   Due to August 4.