Math 20B, Summer-July-2015.
HOMEWORK ASSIGNMENT

During the course, the HW assignment may be slightly changed. ***Please, watch the current assignment***.

In problems marked by *, the student may skip the final calculations, stopping at the moment when the rest calculations are obvious.
Say, $\int_0^{\pi/2} \cos x(1+3\sin^{100} x)^2 dx = \int_0^1 (1+3u^{100})^2 du$ (say why!). If such a problem marked by *, you do not have to (though, certainly, may) compute the last integral but you should know how you would do that. (By the way, how would you do that in this particular case?)
Other examples: $\int \frac{dx}{x\ln^{100} x} = \int \frac{du}{u^{100}}$, where $u = \ln x$. If such a problem marked by *, you do not have to spend your time on finishing the last integral.
Or $\int_2^7 3x^5 dx = \frac{3}{6}x^6|_2^7$. You may stop at this moment if the problem marked by *.

1. **5.2** (Please, use the geometry argument): 4, 6, 7, 10*.

   5.3: 23*, 24*, 33, 35, 55 (yes, use the 1st FTC).

   5.4: *Preliminary questions:* 3. *Exercises:* 7, 14, 19, 23, 25, 29, 31, 35, 41 (important!, may be in a quiz), 42

   5.5: 2*, 13, 22ab.

   5.6: 7, 10, 15, 27, 29, 34, 46 (set $u = x^2 - 1$), 69, 70.

   Find $\int_0^5 \sqrt{25 - x^2} dx$ by substitution $x = 5\sin t$; realize that you are computing a quarter of the area of a disk with a radius of 5.

   77, 80* , 81* (*Hint*: the denominator contains x^2, while the numerator contains x), 83* (*Hint*: the first factor contains x, while the second contains x^2) , 84*, 89*, $\displaystyle\int_1^2 \frac{\ln x}{x} dx$ (you also do not have to calculate the final answer).

   Due to July 6.

2. **6.1**: 9*, 12* , 13*, 15*.

   6.2: 5 (look how we computed the volume of the sphere. What is the difference?) 39, 44 (*Advice*: Use the variable change $u = 1/x$), 61 (just give a rough sketch).

6.3: 1, 11, 21* (just write an integral representing the volume).

Due to July 10.

3. **11.3**: 3cd, 5bc, 13, 21, 24, 27 (sketch the graph, consider all θ from 0 to ∞.

   An additional (and important) problem: Proceeding from the graph of the cardioid $r = 1+\cos\theta$, sketch the graphs of the curves $r = 2+\cos\theta$ and $r = 1 + \sin\theta$.

   Optional problems: 1) Show that the equations $r = \cos 2\theta$ and $r = |\cos 2\theta|$ describe the same curve: a four-petaled rose. 2) What is the difference between curves $r = |\cos 2\theta|$ and $r = |\sin 2\theta|$? (Just sketch the graphs.)

   11.4: 1, 10, 11.

4. **Complex numbers**: *Supplement*: **p.12**: 1b, 2, 3ab, 4abc.

   p.15: 1a, 3, 4; **p.18**: 1, 2, 3, 4 (Regarding the last two problems one can solve them in a straightforward fashion, or do them simultaneously writing

   $\int e^{-7x}\cos 2x dx + i\int e^{-7x}\sin 2x dx = \int e^{-7x}e^{i2x}dx = \int e^{-7x+i2x}dx.$

   One more problem: Find $\int e^x \cos^2 x dx$. (You do not have to complete the problem; just make sure that you can do it. *Advice*: Use a trigonometric formula.)

   Due to July 15.

5. **7.1**: 1, 2, 9,11, 20, 23, 59*.

6. **7.2**: 1, 5, 9 (you may use the formula $\cos^2 x = \frac{1+\cos 2x}{2}$; a similar remark concerns the next problem 11), 11, 26*, 28*, 48, 57 (use the formula for $\sin x \cos y$).

   7.3: 6, 7, 15.

   7.5: 1a, 5, 9, 13, 49. In addition, compute the integrals

   $$\int \frac{x^2}{(x-1)(1+x^2)} \text{ and } \int \frac{x^2}{9+x^2}.$$

   7.6: 1 (you do not have to write answers; just do it (but do it indeed) for yourself), 5, 6, 13, 21, 32, 35, 36, 39-40 (try to guess whether the

integrals converge without any operations: just looking at the order of the functions); 47.

Due to July 24.

7. **10.1**: 15, 16, 17, 21, 40, 43, 54, 57, 61, 62, 63.

   Find the limits of the following sequences:

   (a) $a_n = \dfrac{\ln n}{n}$. (*Advice*: You may use L'Hôpital's rule.)

   (b) $a_n = \dfrac{\ln n}{n^\alpha}$ for any (arbitrary small) $\alpha > 0$. (*Advice*: You may use L'Hôpital's rule)

   (c) $a_n = \dfrac{\ln^{100} n}{n}$. (*Advice*: Write $a_n = \left(\frac{\ln n}{n^{1/100}}\right)^{100}$, and use the previous exercise.)

   (d) $a_n = n^{1/n}$. (*Advice*: Consider $b_n = \ln a_n$, and use an exercise above.)

   (e) $a_n = \dfrac{n}{n + n^{1/n}}$.

   10.2: 11, 17, 18, 22.

   10.3: 1, 2, 10.

   In the next four problems, you do not have to write detailed answers. However, make sure that you know how to justify them. 17, 21, 24. 28.

   In the following problems, figure out whether the series converge applying any criterion you like. You do not have to write detailed solutions. However, say which criterion you use, and make sure that you know how to justify your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{100}}$;

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/100}}$;

   (c) $\displaystyle\sum_{n=11}^{\infty} \frac{1}{\sqrt{n-10}}$;

(d) $\displaystyle\sum_{n=11}^{\infty} \frac{1}{n-10}$;

(e) $\displaystyle\sum_{n=11}^{\infty} \frac{1}{(n-10)^2}$;

(f) $\displaystyle\sum_{n=1}^{\infty} e^{-n}$;

(g) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$ (*Advice*: Compare with $\sum_{n=1}^{\infty} e^{-n/2}$.)

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$

(i) $\displaystyle\sum_{n=1}^{\infty} \sin^2(1/n)$ (*Advice*: Compare with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.);

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n+n}{e^{2n}-n^2}$;

(k) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n+n}{e^{2n}-n^{100}}$.

8. **10.4**: *Preliminary question*s: 1. *Exercises*: In the following problems, figure out whether the series converge. You do not have to write detailed solutions, but make sure that you can justify the answers. 3, 7, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}}$, $\sum_{n=1}^{\infty} \frac{\sin(n\pi)}{\sqrt{n}}$, 9, 19, 22.

   10.5: In the following problems, figure out whether the series converge. You do not have to write detailed solutions, but make sure that you can justify the answers. 1, 5, 7, 21, 25.

   Due to July 29 .

9. **10.6**: In each of the problems below, it is very useful first to guess the answers: 5, 7, 9, 10, 13, 14,25, 29, 45.

   10.7: Preliminary question: 1, Problems: 1, 3, 4, 5, 8, 15 16, 38.

   Due to July 31 .