MATH 120 **FINAL** **December 9, 1998**

1. Compute the following integral; justify your steps.

$$\int_{-\infty}^{\infty} \frac{\sin 4x}{x^2+6x+25}\, dx$$

2. Compute the Laurent series of $\frac{1}{z+2}$ in powers of $z+3$ which converges for $z=5$; moreover, describe the maximum annular region in which this series converges.

3. Compute the following residues at $z=0$:

$$(a) \quad \frac{1-\cosh z}{z^3} \qquad\qquad (b) \quad \frac{e^z-1}{z}$$

4. Let $f(z)=z^3-3z^2+5$

   (a) Compute the values z_0 for which $f'(z_0)=0$

   (b) Determine the values z_1 (if any) for which $|f(z_1)|$ has a relative maximum, i.e. for which $|f(z_1)| \geq |f(z)|$ for all z in a neighborhood U of z_1. Justify your answer.

5. (a) Compute the integral $\int_C \frac{1}{z^{10}+1}\, dz$ where C is the square of sidelength 2.2, centered at the origin (see below).

   (b) Compute the integral $\int_{\tilde{C}} \frac{1}{z^{10}+1}\, dz$ where $\tilde{C}$ is obtained by changing C near i as indicated below. You may use the identity $z^{10}+1=(z^2+1)(z^8-z^6+z^4-z^2+1)$, if needed.

6. Explain why the following integrals do or do not make sense as stated. In each case in which the integral makes sense, compute it.

$$(a) \quad \int_3^5 \frac{z+i}{z-i}\, dz \qquad\qquad (b) \quad \int_i^2 2z\, dz$$

7. Determine all entire functions f for which $|f'(z)| \leq 10$, for all z.