Math 102
Winter 2008
Homework #5

$\boxed{3.1}$

(2) $\left\{ \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}2\\5\end{pmatrix} \right\}$ is a $\underbrace{\text{L.I. set}}$ that is $\underbrace{\text{not orthogonal}}$

$c_1\begin{pmatrix}1\\1\end{pmatrix} + c_2\begin{pmatrix}2\\5\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ only if $c_1 = c_2 = 0$ $\qquad (1\ 1)\begin{pmatrix}2\\5\end{pmatrix} = 1\cdot 2 + 1\cdot 5 = 7 \neq 0.$

$\left\{ \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix} \right\}$ is an $\underbrace{\text{orthogonal set}}$ that is $\underbrace{\text{not L.I.}}$

$(1\ 1)\begin{pmatrix}0\\0\end{pmatrix} = 0 + 0 = 0 \qquad 0\begin{pmatrix}1\\1\end{pmatrix} + c\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$ for all $c \in \mathbb{R}$!

(6) $(x\ y\ z)\begin{pmatrix}1\\1\\1\end{pmatrix} = 0 \Rightarrow x + y + z = 0$

$(x\ y\ z)\begin{pmatrix}1\\-1\\0\end{pmatrix} = 0 \Rightarrow x - y + 0 = 0$

This is what it means for $\begin{pmatrix}x\\y\\z\end{pmatrix}$ to be orthogonal to both $\begin{pmatrix}1\\1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\0\end{pmatrix}$.

So all vectors orthog to both and must satisfy

$$\begin{array}{l} x+y+z=0 \\ x=y \end{array} \Rightarrow \begin{array}{l} z=-x-y=-2y \\ x=y \end{array} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}y\\y\\-2y\end{pmatrix} = y\begin{pmatrix}1\\1\\-2\end{pmatrix}$$

So the subspace in question has basis $\left\{ \begin{pmatrix}1\\1\\-2\end{pmatrix} \right\}$

It only has 1 vector, so it's trivially an orthogonal basis; to make it orthonormal, we need only normalize the vector:

$$\left\| \begin{pmatrix}1\\1\\-2\end{pmatrix} \right\| = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{6}$$

$\therefore$ an O.N.B. for the subspace is $\boxed{\left\{ \begin{pmatrix}1/\sqrt{6}\\1/\sqrt{6}\\-2/\sqrt{6}\end{pmatrix} \right\}}$.