

**MATH. 104A, HOMEWORK 1 (DUE 10/4)**

Do the following questions from the textbook:

Sect. 1.2: # 2, 4, Sect. 1.2 (Miscellaneous Ex.): # 1, 7, 8

In addition, do the following problems:

**(1)** Using mathematical induction, show the following statements:

(a) for all integers  $n \geq 1$ ,

$$1^3 + 2^3 + \dots + n^3 = \left( \frac{n(n+1)}{2} \right)^2.$$

(b) for all integers  $n \geq 5$ ,  $2^n > n^2$ .

**(2)** If  $d$  divides  $ab$ , is it always the case that  $d$  divides  $a$  or  $d$  divides  $b$ ? Justify your answer.

**(3)** We showed in class that if  $n$  is odd, then  $n^2$  is of the form  $4k+1$ . Show that in fact, if  $n$  is odd,  $n^2$  is of the form  $8k+1$ .

**(4)** Assuming the result in (3), show that if  $n$  is of the form  $8k+7$ , then one cannot find integers  $x, y, z$  such that

$$n = x^2 + y^2 + z^2.$$

**(5)** Find all primes of the form  $n^3 - 1$ , with  $n \in \mathbb{Z}$ . (Hint: recall the factorization  $x^3 - 1 = (x - 1)(x^2 + x + 1)$ .)