MATH. 104A, HOMEWORK 2 (DUE 10/11)

Do the following questions from the textbook:

Sect. 2.1: # 2, 6, 8, 12; Sect. 2.2: 14

In addition, do the following problems:

(1) Show that two consecutive integers are relatively prime.

(2) If a, b and c are 3 integers such that $GCD(a, b, c) = 1$, is it necessarily true that they are pairwise relatively prime? Justify your answer.

(3a) If a and b are two non-zero integers, consider the set

$$S = \{c \quad \text{positive} : a|c \quad \text{and} \quad b|c\}.$$

This is the set of **common multiples** of a and b. Why does this set has a smallest element? The smallest element in S is called the **least common multiple** of a and b. It is denoted by $LCM(a, b)$.

(b) Show that if c is such that $a|c$ and $b|c$, then $LCM(a, b)$ divides c. In other words, $LCM(a, b)$ divides any common multiple of a and b. (Hint: apply division algorithm to the numbers c and $LCM(a, b)$).

(c) If we have the prime factorization of a and b:

$$a = p_1^{a_1} \ldots\ldots p_r^{a_r} \quad \text{and} \quad b = p_1^{b_1} \ldots p_r^{b_r},$$

what is the prime factorization of $LCM(a, b)$? Justify your answer.

(4a) We showed in class that if p is prime, then $p|ab \Longrightarrow p|a$ or $p|b$. Using mathematical induction (on k), show that if p is prime, then

$$p|a_1 a_2 \ldots a_k \Longrightarrow p|a_i \quad \text{for saome } i.$$

(b) Show that if p is prime, then $p|a^n$ implies $p^n|a^n$.