MATH. 104A, HOMEWORK 5 (DUE 11/1)

Do the following questions from the textbook:

Sect. 3.2: # 17, 18, 31

Sect. 3.3: 3, 4, 8, 21

In addition, do the following problems:

(1) Given an integer n, let us write n in its base-10 expansion: $n = a_k a_{k-1} .... a_0$ (so $0 \leq a_i \leq 9$). We showed in class that an integer is divisibly by 9 if and only if the sum of the a_i's is divisible by 9. Show that n is divisible by 11 if and only if

$$(a_0 + a_2 + a_4 + ...) - (a_1 + a_3 + a_5 + ...)$$

is divisble by 11.

(2) Find the remainder when

(a) 42^{92093} is divided by 43;

(b) 89^{45} is divided by 43.

without using Fermat's little theorem. Redo Part (b) using Fermat's little theorem.

(3) Show that if n is composite and $n > 4$, then $(n-1)! \equiv 0(\mod n)$. (In particular, the converse of Wilson's theorem holds: n not prime implies that $(n-1)! \neq -1(\mod n)$).