

MATH. 104A, HOMEWORK 7 (DUE 11/15)

Do the following questions from the textbook:

Sect. 2.4: # 6, 7, 8

Sect 3.6: # 2, 4, 16

In addition, do the following problems:

(1) If f is a multiplicative function which is not identically zero, show that $f(1)$ must be equal to 1.

(2) Define a function on positive integers by

$$\mu(n) = \begin{cases} 1, & \text{if } n = 1; \\ (-1)^k, & \text{if } n = p_1 p_2 \dots p_k \text{ is square-free;} \\ 0, & \text{if } n \text{ is not square-free.} \end{cases}$$

This is called the Mobius function.

(a) Find the value of $\mu(30)$, $\mu(35)$ and $\mu(54)$.

(b) Show that μ is multiplicative.

(c) Find the function g defined by

$$g(n) = \sum_{d|n} f(d).$$

(d) For any positive integer n , show that there exist n consecutive integers on which μ takes the value 0. (Hint: think about Chinese Remainder Theorem).

(3) Find all n such that $\sigma(n) = 28$.