

MATH. 140A, SAMPLE MIDTERM 1

You have **50 minutes** for this exam. Please write legibly and show all working. **No calculators are allowed.** Write your name and ID number below.

**Name:**

**ID Number:**

(1) (10 points) Let  $\alpha$  and  $\beta$  be two positive real numbers which are  $> 1$ . Show that there exists a positive integer  $N$  such that  $\alpha^N > \beta$ . You should only use the fact that  $\mathbb{R}$  is an ordered field with the least upper bound property.

(2) (10 points) State and prove the Cauchy-Schwarz inequality. When does equality hold in the Cauchy-Schwarz inequality?

(3) (10 points) Decide if the following are true or false. Justify your answers.

(a) In an ordered field  $A$ , it is always true that  $x^2 > 0$ .

(b) If  $A$  is a countable set and  $B$  is an uncountable set, then  $A \cup B$  is necessarily uncountable and  $A \cap B$  is necessarily countable.

(c) If  $A$  is uncountable and  $J$  is a finite set, then  $A$  has the same cardinality as  $A \cup J$ .