MATH. 140A SAMPLE MIDTERM 2

You have **50 minutes** for this exam. Please write legibly and show all working. **No calculators are allowed.** Write your name and ID number.

Name:

ID Number:

(1) (10 points) Give the definitions of the following terms:

(a) a metric space;

(b) an open subset of a metric space;

(c) a dense subset of a metric space.

Show that $\mathbb{R}^n$ has a countable dense subset.

(2) (11 points)
(a) Show that a compact subset of a metric space is necessarily closed and bounded.

(b) Give an example to show that the converse of (a) is not true.

(c) Show that a closed subset of a compact set is necessarily compact.

(3) (9 points) Decide if the following statements are true or false, giving justifications to your answers.

(a) A countable subset of a metric space X is disconnected.

(b) $(1, 2]$ is not compact.

(c) The limit of a sequence x_n in a metric space is a limit point of the set $\{x_n : n \in \mathbb{Z}^+\}$.