MATH. 200C, HOMEWORK 3

Do the following questions from the textbook:

Sect. 14.2: # 4, 6*, 8*, 14, 15, 17, 23, 24*, 25*, 26*, 27

Sect. 13.4: # 4, 5*, 6*

Sect. 13.5: # 5

In addition, do the following problems:

(1) Let p_1,...,p_r be distinct prime numbers and let $K = \mathbb{Q}(\sqrt{p_1}, ..., \sqrt{p_r})$.

(a) Show that $K/\mathbb{Q}$ is Galois and find its Galois group.

(b) Show that $K = \mathbb{Q}(\sqrt{p_1} + ... + \sqrt{p_r})$.

(3a) Let p be a prime number. Find an irreducible polynomial f of degree p over $\mathbb{Q}$ which has exactly 2 non-real roots in $\mathbb{C}$. (Hint: try a polynomial of the form $x^p - (mp)^3 x(x-1)(x-2)...(x-(p-4)) - p$ for so,me large m coprime to p.)

(b) Show that the splitting field of f has Galois group S_p.

(c) Deduce that for any finite group G, there exists field extensions $\mathbb{Q} \subset F \subset K$ such that K/F is Galois with Galois group G.

The Inverse Problem of Galois theory asks whether for any given finite group G, there is a Galois extension of $\mathbb{Q}$ with Galois group G. It is still an unsolved problem.