MATH. 20D, SAMPLE MIDTERM 2

You have **45 minutes** for this exam. Please write legibly and show all working. **No calculators are allowed.** Write your name, ID number and your TA's name below.

Name:

ID Number:

TA's name:

(1) Consider the differential equation

$$y'' + 2ay' + (a^2 + 1)y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

(a) Find the solution.

(b) Describe its behaviour as $t \to \infty$, taking care to distinguish different possibilities based on the value of a.

(2) Consider the differential equation

$$t^2y'' - t(t+2)y' + (t+2)y = 0.$$

(a) Show that $y_1(t) = t$ is a solution.

(b) Using the method of reduction of order, find a second solution.

(c) Show that the two solutions above are linearly independent.

(3) Find the general solution to the following non-homogeneous equation:

$$y'' - 3y' + 2y = g(t).$$

when

(a) $g(t) = e^t$.

(b) $g(t) = t^2$.