

MATH. 20D, SAMPLE MIDTERM 3

You have **45 minutes** for this exam. Please write legibly and show all working. **No calculators are allowed.** Write your name, ID number and your TA's name below.

Name:

ID Number:

TA's name:

(1a) Are the following 3 vectors in \mathbb{R}^3 linearly independent? Justify your answer.

$$v_1 = (1, 2, 3), \quad v_2 = (3, 4, 5) \quad v_3 = (1, 0, 1).$$

(b) Using Gaussian elimination, find all x_1 , x_2 and x_3 such that

$$x_1v_1 + x_2v_2 + x_3v_3 = (1, 0, 0).$$

(2) Consider the differential equation

$$y^{(3)} - 2y^{(2)} + t^{-1}y' - t^2y = 0$$

(a) Show how to express this equation as a system of linear first order equations.

(b) Can the function $t^2 - 1$ be the Wronskian of a set of fundamental solutions of the system of equations in (b)? Explain your answer.

(3) Find the solution to the equation:

$$x'(t) = A \cdot x(t),$$

with initial condition

$$x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

in the following two cases. In each case, give a sketch of the trajectory of $x(t)$ in the phase plane.

(a)

$$A = \begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}.$$

(b)

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}.$$