

MATH. 20D, MIDTERM 3

You have 45 minutes for this exam. Please write legibly and show all working. No calculators are allowed. Write your name, ID number and your TA's name below.

Name: Solutions

ID Number:

TA's name:

(1) Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix}.$$

(a) (10 points) Calculate the determinant of A and find its inverse.

(b) (20 points) Find the eigenvalues and eigenvectors of A .

(c) (25 points) Consider the differential equation:

$$x'(t) = A \cdot x(t).$$

Find the general solution.

$$(a) \det \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix} = (1)(2)(1) + 0 + 0 - 0 - 0 - 0 = \boxed{2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R2/2} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R1-R2} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} R1+R3 \\ R2-2R3 \end{array} \xrightarrow{\sim} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/2 & 1 \\ 0 & 1 & 0 & 0 & 1/2 & -2 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \Rightarrow \boxed{A^{-1} = \begin{pmatrix} 1 & -1/2 & 1 \\ 0 & 1/2 & -2 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$(b) \text{ eigenvalues: } \det(A - rI) = \begin{vmatrix} (1-r) & 1 & 1 \\ 0 & (2-r) & 4 \\ 0 & 0 & (1-r) \end{vmatrix} = (1-r)^2(2-r)$$

so $\boxed{r=2}$ and $\boxed{r=1}$ (with multiplicity 2)