MATH. 20D SAMPLE FINAL (WINTER 2010)

You have **3 hours** for this exam. Please write legibly and show all working. **No calculators are allowed.** Write your name, ID number and your TA's name below. The total number of points for this exam is 180.

Name:

ID Number:

TA's name:

Question	Score
1. (/25)	
2. (/20)	
3. (/35)	
4. (/40)	
5. (/30)	
6. (/30)	
Total (/180)	

(1) (25 points) Find bases for the row space, column space and nullspace of the matrix

$$A = \begin{pmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{pmatrix}$$

(2) (20 points) Give the definitions of the following terms:

(i) a linearly independent set of vectors;

(ii) the nullspace of a matrix A.

(iii) a basis of a vector space

(iv) the dimension of a vector space

(v) the orthogonal complement of a subspace W of $\mathbb{R}^n$.

(3i) (20 points) Find the eigenvalues and associated eigenvectors of the following matrix

$$A = \begin{pmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}.$$

(ii) (10 points) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

(iii)(5 points) Compute A^{10}. You may express your answer as the product of not more than 3 matrices.

(4i) (20 points) LEt W be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$w_1 = (1, -4, 0, 1)^T \quad \text{and} \quad w_2 = (7, -7, -4, 1)^T.$$

By performing the Gram-Schmidt process to these vectors, find an orthogonal basis for W.

(ii) (10 points) Find the distance of y to W, where

$$y = (1, 0, 0, 1)^T$$

(iii) (10 points) Find a basis for $W^{\perp}$.

(5i) (5 points) Let V be the vector space of polynomials of degree ≤ 2. Show that

$$\mathcal{B} = \{t^2 + 1, t - 2, 1\}$$

is a basis of V

(ii) (10 points) Find the coordinate vector of $q(t) = 3t^2 - 1$ with respect to the basis $\mathcal{B}$.

(iii) (5 points) If $T : V \longrightarrow V$ is the function defined by

$$T(p) = p''(t) - tp'(t) + p(0).$$

Show that T is linear.

(iv) (10 points) Find the matrix representing T with respect to the basis $\mathcal{B}$.

(6) (30 points) Determine if the following statements are true or false and give justifications for your answers.

(a) If A is a 4×5 matrix, then there is a vector b such that $Ax = b$ has unique solution.

(b) If A is a 7×5 matrix, then its row space may have dimension 7.

(c) If A is a 6×5 matrix whose row space has dimension 4, then its nullspace has dimension 2.

(d) If A is a 3×5 matrix, then its nullspace must have dimension at least 2.

(e) If AB is invertible, so are A and B. Here, A and B are $n \times n$ matrices.

(f) If A is an orthogonal matrix, then $\det A = 1$ or -1.