MATH. 20F SAMPLE MIDTERM 1

You have **50 minutes** for this exam. Please write legibly and show all working. **No calculators are allowed.** Write your name and ID number.

Name:

ID Number:

(1) Let

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & a & 0 \\ 2 & 1 & 3 \end{pmatrix}.$$

(i) (8 points) Find all a's such that the homogeneous system $Ax = 0$ has non-trivial solutions.

(ii) (7 points) For those values of a from (i), find the general solution of $Ax = 0$.

(2i) (5 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. If x and y in $\mathbb{R}^n$ satisfies

$$T(x) = 0 = T(y),$$

show that $T(x + y) = 0 = T(\lambda x)$ for any scalar λ.

(ii) (5 points) If T is one-to-one, show that $n \leq m$.

(3) (15 points) Find the inverse of the matrtix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 2 & 1 & 1 \end{pmatrix},$$

and express A as a product of elementary matrices.

(4) (10 points) Decide if the following statements are true or false, giving justifications to your answers.

(a) If A is an $n \times n$ matrix such that $A^2 = A \cdot A = 0$, then $A = 0$.

(b) Any collection of 3 vectors in $\mathbb{R}^3$ such that none is a multiple of another must be linearly independent.