

**MATH. 20F SAMPLE MIDTERM 2**

You have **50 minutes** for this exam. Please write legibly and show all working. **No calculators are allowed.** Write your name and ID number.

**Name:**

**ID Number:**

**(1i)** (5 points) Find the determinant of

$$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 11 & 3 & 7 & 5 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

**(ii)** (5 points) If  $A$  is an  $n \times n$  matrix, how are  $\det(3A)$  and  $\det(A)$  related?

(2i) (10 points) Let  $\mathbb{P}_2$  be the vector space of polynomials of degree  $\leq 2$ , and let

$$W = \{p \in \mathbb{P}_2 : p(0) + p'(1) = 0\}.$$

Show that  $W$  is a subspace of  $\mathbb{P}_2$  and find a basis for  $W$ .

(ii) (10 points) Consider the map  $T : \mathbb{P}_2 \rightarrow \mathbb{P}_2$  defined by

$$T(p) = p''(t) + p(0).$$

Show that  $T$  is linear and determine its range.

(3) Let  $\{e_1, e_2\}$  be the standard basis of  $\mathbb{R}^2$ . Consider the following two bases of  $\mathbb{R}^2$ :

$$\mathcal{B} = \{e_1 + e_2, 2e_2\} \quad \text{and} \quad \mathcal{C} = \{e_1 - e_2, e_1\}.$$

(i) (5 points) Find the change of coordinate matrix from  $\mathcal{B}$ -coordinates to  $\mathcal{C}$ -coordinates.

(ii) (5 points) If a vector  $w$  is such that  $[w]_{\mathcal{B}} = (1, 3)^T$ , what is  $[w]_{\mathcal{C}}$ ?

(4) (10 points) Decide if the following statements are true or false. Justify your answers.

(a) If  $A$  is a  $6 \times 7$  matrix whose row space has dimension 2, then the null space of  $A$  has dimension 5.

(b) If  $A$  and  $B$  are related by a sequence of ERO's, then the column space of  $A$  and  $B$  are equal.