14.1 Stein’s method

Stein(1972): Sums of dependent variables, dealing with the error estimate for

\[
\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \xi_i - W(1)
\]

where \(\xi_i\) are identically distributed with mean 0 and variance 1.

Recall that we have \(\bar{X}^r(\cdot) \approx \tilde{X}(\cdot)\) (Process level approximation). We would like to know when \(t \to \infty\), how close is \(\bar{X}^r(\infty)\) and \(\tilde{X}(\infty)\).


14.2 Erlang-A process

We denote Erlang-A process as M/M/n + M queuing system. People coming in the system with rate \(\lambda\), with \(n\) service desks each with service rate \(\mu\). Meanwhile there is an abandonnment rate \(\alpha\) (people waiting in line have a maximum waiting time distributed exponentially with parameter \(\alpha\). When the waiting time of the person in queue exceeds their maximum waiting time, the person leaves the system). Erlang-C process is corresponding to the case when \(\alpha = 0\), or people never abandon the queue.

Denote \(X(t)\) as the total number of people in the system at time \(t\). The state space is \(\mathbb{N}\). It is a birth-death process with birth rate \(\lambda\), and death rate \(\alpha(X(t) - n)^+ + \mu \min(n, X(t))\). The first part is from abandoning from queue, and the second part is from service.

To relate this process to the chemical reaction system, denote the birth process as reaction \(\emptyset \xrightarrow{\lambda} S_1\), the abandoning process as \(S_1 \xrightarrow{\alpha} \emptyset\). The process of service is described as \(S_1 + E \xrightarrow{\eta} S_1E\) (\(\eta\) equal to infinity to make this process instantaneous), and \(S_1E \xrightarrow{\mu} E\).