MATH289: Topic in Probability **Fall 2017**

Lecture 14: November 28

Lecturer: Prof. Ruth Williams *Scribe: Yuchao Liu*

14.1 Stein's method

Stein(1972): Sums of dependent variables, dealing with the error estimate for

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}\xi_i - W(1)$$

where ξ_i are identically distributed with mean 0 and variance 1.

Recall that we have $\bar{X}^r(\cdot) \approx \tilde{X}(\cdot)$ (Process level approximation). We would like to know when $t \to \infty$, how close is $\bar{X}^r(\infty)$ and $\tilde{X}(\infty)$.

Ref: Braverman, Dai, Feng: Stein's method for steady state diffusion approximations: an introduction through the Erlang-A and Erlang-C models. Stochastic Systems 2017. (https://arxiv.org/abs/1512.09364)

See also: Braverman, Dai: Stein's method for steady-state diffusion approximations of M/Ph/n + M systems. Annals of Applied Probability. 27(1): 550-581. (https://arxiv.org/abs/1503.00774)

14.2 Erlang-A process

We denote Erlang-A process as M/M/n + M queuing system. People coming in the system with rate λ, with n service desks each with service rate μ. Meanwhile there is an abandonnment rate α (people waiting in line have a maximum waiting time distributed exponentially with parameter α. When the waiting time of the person in queue exceeds their maximum waiting time, the person leaves the system). Erlang-C process is corresponding to the case when $\alpha = 0$, or people never abandon the queue.

Denote $X(t)$ as the total number of people in the system at time t. The state space is $\mathbb{N}$. It is a birth-death process with birth rate λ, and death rate $\alpha(X(t) - n)^+ + \mu \min(n, X(t))$. The first part is from abandoning from queue, and the second part is from service.

To relate this process to the chemical reaction system, denote the birth process as reaction $\emptyset \xrightarrow{\lambda} S_1$, the abandoning process as $S_1 \xrightarrow{\alpha} \emptyset$. The process of service is described as $S_1 + E \xrightarrow{\eta} S_1E$ (η equal to infinity to make this process instantaneous), and $S_1E \xrightarrow{\mu} E$.

A survey reference: Fundamentals of Stein's method, Nathan Ross, Probab. Surveys Volume 8 (2011), 210-293. (https://arxiv.org/abs/1109.1880).