Jason Fulman's 5 minute presentation on relation of math of Persi Diaconis to his work

Let $d(\pi)$ be the number of descents of a permutation $\pi \in S_n$ – i.e. the number of i with $1 \leq i \leq n-1$ and $\pi(i) > \pi(i+1)$.

Theorem 0.1. *(Bayer and Diaconis) Cut a deck of n cards at position k with probability $\frac{\binom{n}{k}}{2^n}$. Perform one of the $\binom{n}{k}$ interleavings at random. Iterate r times. Then the chance of π is*

$$\frac{\binom{n+2^r-d(\pi^{-1})-1}{n}}{2^{rn}}.$$

This result was very influential: refined work of Aldous on total variation distance, first example of cut-off phenomenon for nonreversible Markov chains, front page of New York Times.

Theorem 0.2. *(Diaconis, McGrath, Pitman) Let π be as in Theorem 0.1. Then the chance that the cycle structure of π is $(1^{n_1}2^{n_2}...)$ is equal to the chance that a random degree n monic polyomial with coefficients in F_{2^r} has factorization type $(1^{n_1}2^{n_2}...)$.*

Seeing Theorem 0.2, I became intersted in connections to Lie theory. This is because degree n polynomials are the semisimple orbits of the adjoint action of $GL(n,q)$ on its Lie algebra, and in Lie theory there is a natural map from semisimple orbits to conjugacy classes of the Weyl group. It turns out to be better to use SL than GL–then the result doesn't hold for all primes but generalizes better to other types. I found a type B analog, but the general picture is still not entirely clear. For details, see my paper in J. Algebra Volume 224.

This unsatisfactory state of affairs led me to look for a better theory. Cellini defined certain elements in the group algebra of the Weyl group by taking minimal coset representatives in the affine Weyl group of a certain subgroup of the affine Weyl group, and then projecting them to the Weyl group. These elements of the Weyl group have the remarkable property that their powers can be understood, but she didn't find a formula for them in type A. I found such a formula in terms of two well-studied combinatorial parameters: major index of a permutation (sum of positions of descents) and number of cyclic descents of a permutation. For certain values of the parameters, this element of the group algebra corresponds to cut and riffle shuffle. Remarkably (motivated by the way real people shuffle!), Bayer/Diaconis had studied the slightly different situation of shuffle then cut.

With this construction, the connections with Lie theory work in all types and for all primes (using semisimple conjugacy classes instead of semisimple orbits). I had representation theory/combiantorial proofs in some types and conjectured it in all types. This was proved by Roger Carter using the Brauer complex; he also gave an extension for twisted groups. For details, see my J. Algebra papers in Volumes 243, 231 and the references therein. An interesting problem is to explain these results in type A by modifying Gessel's bijection used to prove Theorem 0.2; this seems quite challenging.

This story emphasizes Persi's gifts:

(1) Knows what is interesting before doing computations; doesn't need the crutch of mathematical structure–shuffling , quivers, etc.
(2) Sees deep theories in simple things–Feynman's cup of wine.
(3) Thanks for making math friendly, not intimidating, appealing to people.
(4) Brings out the best in people.