0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Find an equation of the plane containing the two parallel lines

\[ \ell_1(t) = (1, 1, 1) + t(1, 2, 3) \quad \text{and} \quad \ell_2(t) = (1, 2, 3) + t(1, 2, 3). \]

Write your answer in the form \( Ax + By + Cz = D \).

2. (6 points) Find a vector equation for the line of intersection formed by the intersection of the two planes

\[ 2x - y + z = 5 \quad \text{and} \quad x + y - z = 1. \]

3. (6 points) The light intensity (measured in lux) in a certain pond at point \((x, y, z)\) is given by

\[ f(x, y, z) = e^{-x/2} + e^{-y/3} + e^{-z/4}. \]

A South American electric eel is swimming in the pond along a path with position after \(t\) seconds given by the function \(c(t) = (x(t), y(t), z(t))\). Suppose \(c(5) = (1, 0, 2)\) and \(c'(5) = (3, 9\pi, 2)\). How fast is the light intensity increasing along the eel’s path after 5 seconds?

4. (6 points) A Higgs boson (an elementary particle) is moving along a helical path given by the equations

\[ x(t) = 5 \cos(2t), \quad y(t) = t, \quad z(t) = 5 \sin(2t). \]

(a) Find the distance traveled by the particle between \(t = 0\) and \(t = 2\pi\).

(b) Find the displacement between the particle’s position at \(t = 0\) and its position at \(t = 2\pi\).

5. (6 points) A triangle has vertices \(A = (0, 0, 0), B = (1, 2, 3), \) and \(C = (-3, -1, 3)\). Find the three angles of the triangle. Leave your answers in terms of arccos or \(\cos^{-1}\).

(Please turn over.)

(This exam is worth 50 points.)
6. (6 points) The gradient of the function $g(x, y, z)$ at $(−3, 4, 5)$ is the vector $⟨−2, 1, 2⟩$.
   (a) Find the values of $g_x$ and $g_y$ and $g_z$ at $(−3, 4, 5)$.
   (b) Find the maximum rate of change of $g(x, y, z)$ at $(−3, 4, 5)$.
   (c) Find the rate of change of $g$ at $(−3, 4, 5)$ in the direction of the point $(−1, 8, 1)$.

7. (6 points) Evaluate the iterated integral

\[
\int_{y=0}^{2} \int_{x=y/2}^{1} ye^{x^3} dx \, dy.
\]

Change the order of integration, if necessary.

8. (6 points) Lagrangian Airlines limits the size of carry-on bags by requiring the length of the bag plus the girth (see below) to be less than or equal to 90 inches. Find the dimensions of the bag with the largest volume that can be taken on the plane. (Assume the carry on is a rectangular box, like in the picture below.)

![Diagram of a rectangular box with labels for length and girth.](image)

Girth is the distance all the way around the bag moving perpendicular to the length.