Math 281A Homework 4
Due: Oct 31, in class

1. Let \( \{X_i\}_{i=1}^n \) be an i.i.d. sample from Poisson distribution with mean \( \theta \). Find a variance stabilizing transformation for the sample mean and construct a confidence interval for \( \theta \) based on this.

2. Let \( X_1 \sim \text{Uniform}(0,2\pi) \), and let \( X_2 \sim \text{exp}(1) \), independent of \( X_1 \). Find the joint distribution of \( (Y_1, Y_2) = (\sqrt{2X_2}\cos X_1, \sqrt{2X_2}\sin X_1) \).

3. Let \( \{X_i\}_{i=1}^n \) be i.i.d. from logistic distribution with cdf
   \[ F_\theta(x) = \frac{e^{tx}}{1 + e^{tx}}, \text{ for } t \in \mathbb{R}. \]

   (a) Find the asymptotic distribution of \( X_{(n)} - X_{(n-1)} \);

   (b) Based on part (a), construct a 95\% confidence interval for \( \theta \). You can use the fact that 0.025 and 0.975 quantiles of standard exponential distribution are 0.0253 and 3.6889;

   (c) Simulate 1000 samples of size \( n = 40 \) and \( \theta = 2 \). How many confidence intervals contain \( \theta \)?