Math 3C Section 3.1

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1 Function and Domain

A **function** is a rule that describes, given a value of variable x, how to associate it with another unique real number y. For example, $y = x + 1$. The y value is also frequently refer to as "function", like in the statement: y is a function of x. The name of variable and function and is of little importance and contingent on the context. For example, $y = x + 1$ and $z = t + 1$ shall be considered as the same function, used in different situations: dad(y) is one year elder than mom(x), and I(z) am one year elder than my brother(t). We choose different name for variable but they expressed the same relation between ages.

The usual notation for function is $y = f(x)$, where f alone means the mapping rule. We can also think of f as a machine which takes imput x and produces output y. But a machine cannot take all kinds of imput since its capacity is limited. So we have the notion of **domain** of a function.

Domain of a function is the **set** of real numbers that can be taken as imputs by the function.

There are two cases when a function $y = f(x)$ is given:
(1) the domain of f is specified (e.g. $y = 2x + 3$, x in $[-1, 1]$.)
(2) the domain of f is not specified due to laziness (e.g. $y = \sqrt{x-2}$.)

For case (1), we just need to keep in mind that the domain is an equally important part as the expression of f. Functions like

$$f(x) = |x|, x \text{ in } [-1, 1] \quad \& \quad f(x) = |x|, x \text{ is a real number}$$

are completely different functions. For case (2), the principle is: domain is the **largest possible** set of real numbers x so that $f(x)$ can be computed to be a real number. There are 2 frequently tested patterns in exercises: square root and fractional expressions. Let's look at the following sequence of examples:

Example 1. Find the domain of $f(x) = \sqrt{x-2}$.
Solution. Since the square root of a negative number is not a real number, we need $x - 2 \geq 0$. Therefore the domain is $\{x : x \geq 2\}$, or just $[2, \infty)$.

Example 2. Find the domain of $f(x) = 1/(x-2)$.
Solution. As $x-2$ appears on the denominator, it can not be 0. So the domain is $\{x : x \neq 2\}$ or $(-\infty, 2) \cup (2, \infty)$.

Example 3. Find the domain of $f(x) = \sqrt{(x+4)/(x-2)}$.
Solution. First we see the square root and fraction patterns in the function f. The final operation is the square root, which means that

$$\frac{x+4}{x-2} \geq 0.$$

Also we observe that $x - 2 \neq 0$ since it is the denominator. Let us deal with the fraction inequality above. Since we do not know whether $x-2$ is positive or negative, we cannot multiply $x-2$ to both sides and keep the $\geq$ sign (see the multiplicative property of inequalities), therefore we multiply $(x-2)^2$ to both sides. Hence we have

$$\begin{aligned} & \frac{x+4}{x-2} \cdot (x-2)^2 \geq 0 \cdot (x-2)^2 \\ \Longrightarrow \quad & (x+4)(x-2) \geq 0 \end{aligned}$$

To solve the above inequality, we have the following simple method(it is a variation of method of graphs): solve equations $x+4=0$ and $x-2=0$, get $x=-4$ and $x=2$. Find -4 and 2 on the real line. Taking x to be a sufficiently large number, like 100, we have $(100+4)(100-2) > 0$, therefore we start with a point above the real line on the far right. Then we trace back towards the real line and hit it at $x=2$, entering the lower half of the plane. The curve comes back to the real line at -4 which is another zero point. Since there is no more zero point on the left the curve will keep above the line. Observe that on $[2, \infty)$ and $(-\infty, 4]$ the curve is above the line, the solution to the inequality is

$$x \text{ in } [2, \infty) \cup (-\infty, 4]$$

Considering that $x \neq 2$, the domain of f is

$$(2, \infty) \cup (-\infty, 4].$$

Next we talk about the range of a function, which is the set of values of function when the variable varies over the domain. To find the range of a function $y = f(x)$ on some domain, it is sometimes tricky when we only know the explicit formula $f(x)$. A more convenient way is to find the **graph** of f and extract information directly from the graph.

2 Graph of Functions

(*Please see the handwritten notes for this section.*)