Graph of a Function

For a function $y = f(x)$, where x is in some domain D, the graph of f is the collection of points $(x, f(x))$ on the coordinate plane. Following this definition, the procedure of sketching graph of function is as follows.

Sketch the graph of $y = f(x)$

(1). Take sufficiently many "x" values in the domain D, $\{x_1, x_2, \cdots, x_{1000}\}$ and evaluate f at $\{x_1, x_2, \cdots, x_{1000}\}$, and get the following list.

x	x_1	x_2	x_3	$\cdots$	x_{1000}
$f(x)$	$f(x_1)$	$f(x_2)$	$f(x_3)$	$\cdots$	$f(x_{1000})$

(2) Find $(x_1, f(x_1)), (x_2, f(x_2)), \cdots (x_{1000}, f(x_{1000}))$ on the coordinate plane

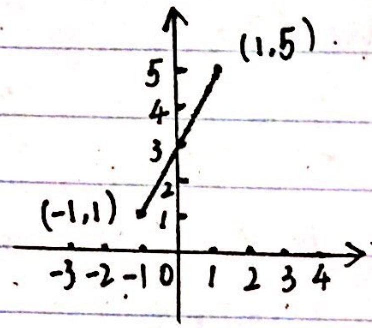
(3) Find a smooth curve which passes through these points.

However, we are not going to use this method to sketch graphs of all functions. For some of simple functions the property of their graphs are well known, and we can sketch them without heavy computation.

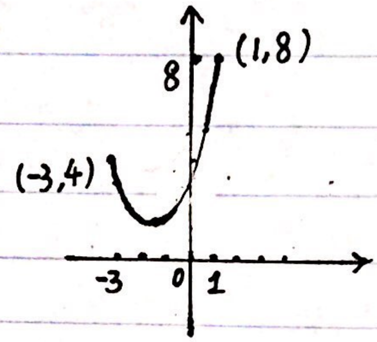
① Linear Function $y = kx + b$.

e.g. $y = 2x + 3$, x in $[-1, 1]$

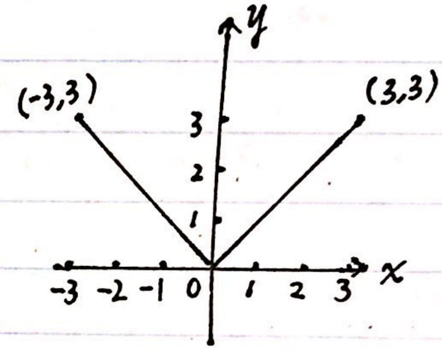
Notice that the end points of the domain is $-1, 1$, and the graph of a linear function is a straight line, we simply evaluate it at -1 and 1. then use a segment to connect $(-1, 1)$ and $(1, 5)$.



graph of $y=2x+3$, $[-1,1]$.



$y=x^2+3x+4$, on $[-3,1]$



$y=|x|$, on $[-3,3]$

② Quadratic Function $y=ax^2+bx+c$

e.g. $y=x^2+3x+4$ x in $[-3,1]$

The graph is generally, a parabola with vertex $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$, and if we find the end points of the domain, we have 3 points. This is enough to sketch the graph.

(1) vertex $\left(-\frac{3}{2}, \frac{7}{4}\right)$.

(2) end points $(-3,4)$, $(1,8)$

(3) connect three points with a smooth parabola.

③ Absolute Value Function $y=|x|$

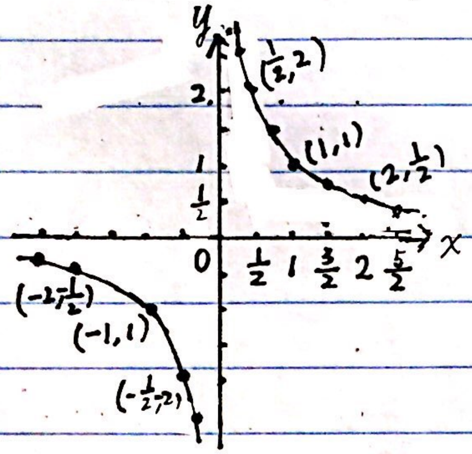
e.g. $y=|x|$, x in $[-3,3]$

The graph is a "V" shaped piecewise linear curve. Using end points $(-3,3)$, $(3,3)$ and vertex $(0,0)$.

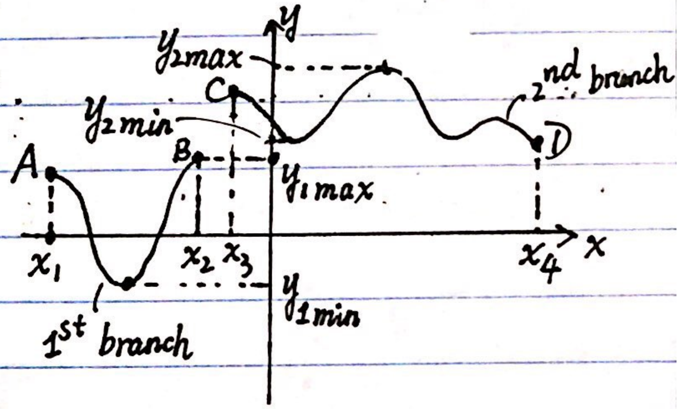
④ Inversely Proportional Function $y=\frac{1}{x}$.

The graph consists of two curves contained in the first quadrant and third quadrant respectively. It is also called a hyperbola.

Another geometric feature of the graph is that both curves approach x and y axis as being extended indefinitely.



$y = \frac{1}{x}$



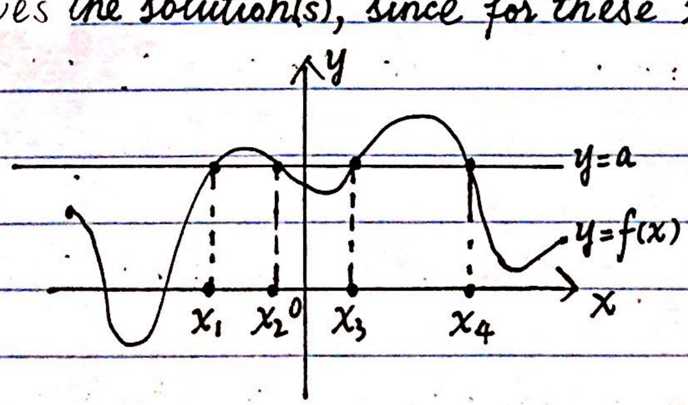
$y = f(x)$, an arbitrary fn.

How to extract information of a function from its graph?

For example, the graph above is made up of 2 branches, hence we choose end points A, B, C, D, drop a perpendicular segment to x-axis, obtain its domain $[x_1, x_2] \cup [x_3, x_4]$. Then we choose points with maximal and minimum height and find the range: $[y_{1min}, y_{1max}] \cup [y_{2min}, y_{2max}]$. To find the solution of an equation

$$f(x) = a$$

find the straight line $y = a$, the intersection of the graph and $y = a$ gives the solution(s), since for these x value, $f(x) = a$ holds.



Solving eqn. $f(x) = a$.