MATH 142A - INTRODUCTION TO ANALYSIS
PRACTICE MIDTERM 1

WINTER 2021

Name (Last, First): _____

Student ID: _____

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT.

1. Prove that for any $n \in \mathbb{N}$

$$(2n)! < 2^{2n}(n!)^2. \tag{1}$$

2. Prove that the set

$$S := \left\{ \frac{n}{n+3}(2 + (-1)^n) \,:\, n \in \mathbb{N} \right\} \tag{2}$$

is bounded. Determine $\sup S$ and $\inf S$ (provide the proof).

3. By checking the definition of a convergent sequence, compute the limit of the sequence $(a_n)_{n=1}^{\infty}$ with

$$a_n = \sqrt{n+1} - \sqrt{n}. \tag{3}$$

4. Determine

$$\lim_{n\to\infty} \Big(\frac{1}{2n} - \frac{2}{2n} + \frac{3}{2n} - \cdots + \frac{2n-1}{2n} - \frac{2n}{2n} \Big). \tag{4}$$

5. Prove that the sequence $(b_n)_{n=1}^{\infty}$ with

$$b_n = 1 + \frac{1}{2 \cdot 2} + \frac{1}{3 \cdot 2^2} + \frac{1}{4 \cdot 2^3} + \cdots + \frac{1}{n \cdot 2^{n-1}} \tag{5}$$

is convergent.