MATH 142A - INTRODUCTION TO ANALYSIS
PRACTICE MIDTERM 2

WINTER 2022

Name (Last, First): ______________________________

Student ID: ______________________________

Clearly indicate all results from Lectures 4-16 that you use in your solutions.

1. Let (s_n) be a monotonic sequence and let (s_{n_k}) be its subsequence. Prove that if the subsequence (s_{n_k}) is a Cauchy sequence, then (s_n) converges.

2. Determine the set of the partial limits, lim inf and lim sup of the sequence (x_n) given by

$$x_n = \frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}. \tag{1}$$

3. Determine if the following series converge

(a)

$$\sum_{n=2}^{\infty} \frac{3}{\log n}$$

(b)

$$\sum_{n=2}^{\infty} \frac{3^n}{(\log n)^n}$$

4. Prove that the function

$$f(x) = 2^{\frac{1}{1+x^2}}$$

is continuous on $\mathbb{R}$.

5. Let $S \subset \mathbb{R}$ and let $f : S \to \mathbb{R}$ and $g : S \to \mathbb{R}$ be uniformly continuous on S. Prove that $f + g$ is uniformly continuous on S.