☐ Write your name and PID on the top of EVERY PAGE.

☐ Write the solutions to each problem on separate pages. CLEARLY INDICATE on the top of each page the number of the corresponding problem. Different parts of the same problem can be written on the same page (for example, part (a) and part (b))

☐ Remember this exam is graded by a human being. Write your solutions NEATLY AND COHERENTLY, or they risk not receiving full credit.

☐ From the moment you access the midterm problems on Gradescope you have 70 MINUTES to COMPLETE AND UPLOAD your exam to Gradescope. Plan your time accordingly.

☐ All steps of the proofs should be INCLUDED in your solutions. Provide references to the theorem/examples from the lectures/textbook used in your proofs.

☐ You are allowed to use the textbook, lecture notes and your personal notes. You are not allowed to use the electronic devices (except for accessing the online version of the textbook) or outside assistance. Outside assistance includes but is not limited to other people, the internet and unauthorized notes.

1. (25 points) Let (a_n), (b_n) and (c_n) be three sequences of real numbers satisfying

$$a_n \leq b_n \leq c_n$$

for all $n \in \mathbb{N}$. Suppose that $\lim a_n = a$, $\lim c_n = c$, where a and c are two real numbers, $a < c$.

Let S denote the set of the *subsequential limits* of (b_n). Prove that $S \subset [a, c]$.

2. (25 points) Let (x_n) be a sequence of real numbers given by

$$x_n = (-1)^n \left(1 + \frac{1}{n}\right)^n + \sin \frac{\pi n}{2}$$

for $n \in \mathbb{N}$. Determine the set of the subsequential limits of (x_n), $\limsup x_n$ and $\liminf x_n$.

3. (25 points) Determine if the series

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$$

converges.

4. (25 points) Prove that the function $f(x) = 7^x$ is not uniformly continuous on $\mathbb{R}$.