

Write your name and PID on the top of **EVERY PAGE**.

Write the solutions to each problem on separate pages. **CLEARLY INDICATE** on the top of each page the number of the corresponding problem. Different parts of the same problem can be written on the same page (for example, part (a) and part (b))

Remember this exam is graded by a human being. Write your solutions **NEATLY AND COHERENTLY**, or they risk not receiving full credit.

From the moment you access the midterm problems on Gradescope you have **60 MINUTES** to **COMPLETE AND UPLOAD** your exam to Gradescope. Plan your time accordingly.

All steps of the proofs should be **INCLUDED** in your solutions. Provide references to the theorem/examples from the lectures/textbook used in your proofs.

You are allowed to use the textbook, lecture notes and your personal notes. You are not allowed to use the electronic devices (except for accessing the online version of the textbook) or outside assistance. Outside assistance includes but is not limited to other people, the internet and unauthorized notes.

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1. (25 points) Prove that for all $n \in \mathbb{N}$

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 2 - \frac{1}{n}. \quad (1)$$

2. (25 points) Prove that

$$\lim_{n \rightarrow \infty} \sqrt[n]{n^2 + n} = 1$$

using only the definition of convergence (i.e., without using any theorems about limits of sequences or examples from lectures/textbook). Clearly indicate how you choose $N(\varepsilon)$ for any $\varepsilon > 0$.

3. (25 points) Compute the limit

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2} \sin(n!)}{n + 1}.$$

Clearly indicate all the statements from the lectures/textbook used to compute the limit.

4. (25 points) Prove that the sequence $(x_n)_{n=1}^{\infty}$ with

$$x_n = \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{2^n}\right)$$

converges.