MATH 10C: Calculus III (Lecture B00)

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Today: Vector-valued functions

Next: Strang 3.2

Week 4:

- homework 3 (due Tuesday, October 18)
- Midterm 1: Wednesday, October 19 (vectors, dot product, cross product, equations of lines and planes)

Velocity and acceleration

Imagine a particle moving (smoothly) through space.

Let

The velocity is the

It describes the

The acceleration is the

Mathematically, the velocity is the derivative

and the accelaration is

The derivatives are computed

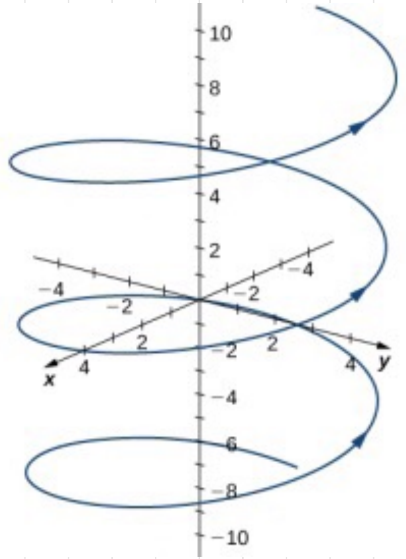
Velocity and acceleration

Example Let $\vec{r}(t) =$

The velocity:

The acceleration:

The path of this particle is called a



Limits of vector-valued functions

Let $\vec{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$ be a vector-valued function and let $\vec{L} = \langle L_1, L_2, L_3 \rangle$. Then the expression

means that

If one or more of the limits $\lim_{t \to t_0} r_1(t)$, $\lim_{t \to t_0} r_2(t)$ or $\lim_{t \to t_0} r_3(t)$ do not exist, we say that $\lim_{t \to t_0} \vec{r}(t)$ does not exist.

Example What is

$$\lim_{t \to 0} \left\langle \frac{\sin t}{t}, e^t, \cos t \right\rangle$$

Continuity of vector-valued functions

A vector-valued function $\vec{r}(t)$ is continuous at t_0 if

This is equivalent to $\lim_{t \to t_0} r_1(t) = r_1(t_0)$, $\lim_{t \to t_0} r_2(t) = r_2(t_0)$ and

$\lim_{t \to t_0} r_3(t) = r_3(t_0)$

Therefore, $\vec{r}(t)$ being continuous at t_0 is equivalent to

We say that $\vec{r}(t)$ is continuous if it is continuous at every single point t_0.

Derivatives of vector-valued functions

The derivative of a vector-valued function $\vec{r}$ is

$$\vec{r}\,'(t) =$$

provided that the limit exists. If $\vec{r}\,'(t)$ exists, we say that $\vec{r}$ is . If $\vec{r}$ is differentiable at every point t from the interval (a,b), we say that $\vec{r}$ is differentiable on (a,b).

Notice that if $\vec{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$, then

$$\vec{r}\,'(t) =$$

$$=$$

Calculus of vector-valued functions

Example Let $\vec{r}(t) = \langle \sin t,\ e^{2t},\ t^2 - 4t + 2 \rangle$

Then

Summary

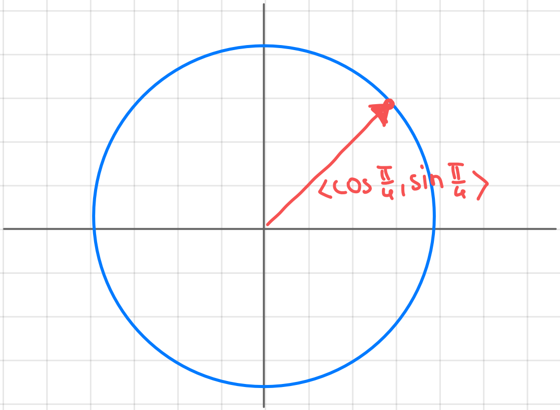
Calculus concepts (limit, continuity, derivative) are applied to vector-valued functions componentwise (apply to each component separately).

If $\vec{r}(t)$ represents the position of some object, then

- $\vec{r}\,'(t)$ is the velocity of this object ($\|\vec{r}\,'(t)\|$ is speed)
- $\vec{r}\,''(t)$ is the acceleration of the object

Tangent vectors. Tangent lines

Let $\vec{r}(t)$ be a vector-valued function. Suppose that $\vec{r}$ is differentiable at t_0.



$\vec{r}(t) = \langle \cos t, \sin t\rangle$

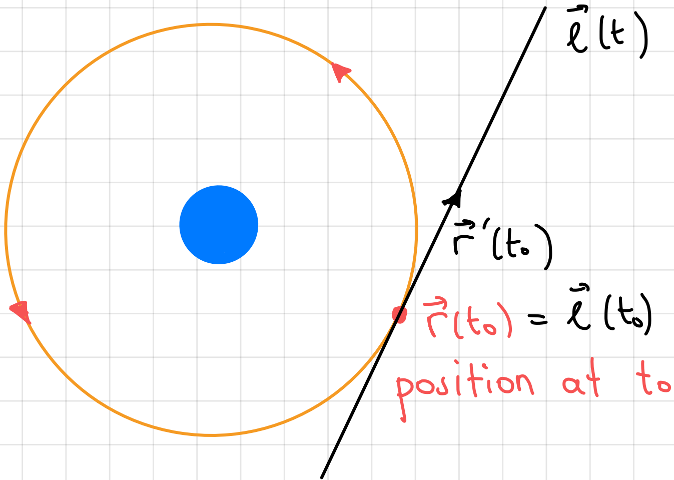
Then vector $\vec{r}'(t)$ is

The targent line to $\vec{r}$ at t_0 is the line given by the vector equation

Tangent vectors. Tangent lines

The tangent line $\vec{\ell}(t)$ to $\vec{r}(t)$ at t_0 has the

Example Imagine satellite orbiting a planet.



If the planet disappears at time t_0, then

Tangent vectors. Tangent lines

Example Let $\vec{r}(t) = \langle t^2 - 2, e^{3t}, t \rangle$

Find the tangent line to $\vec{r}(t)$ at $t_0 = 1$.

First, find the tangent vector at $t_0 = 1$

Next, find the position at $t_0 = 1$

Finally, we can write the equation for the tangent line

Definition We call the principal unit

tantent vector to $\vec{r}$ at t. (provided $\|\vec{r}\,'(t)\| \neq 0$)