MATH 10C: Calculus III (Lecture B00)

mathweb.ucsd.edu/~ynemish/teaching/10c

Today: Directional derivative. Gradient

Next: Strang 4.7

Week 7:

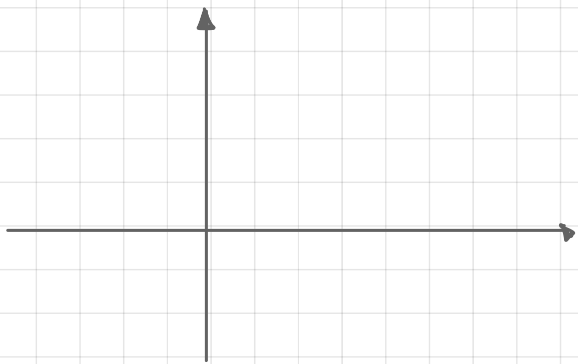
- homework 6 (due Friday, November 11)

Directional derivative

Consider a function of two variables $f(x, y)$.

Then the partial derivatives $f_x(x_0, y_0)$, $f_y(x_0, y_0)$ represent the rate of change of function f at point (x_0, y_0) in the x-direction and in the y-direction correspondingly.

Q: What if we want to know the rate of change in another direction?



Directional derivative

Definition We call

Q: How to compute $D_{\vec{u}} f(x_0, y_0)$?

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Example

Let $f(x,y) = x^2 - xy + 3y^2$. Find the directional derivative of f in the direction $\langle 3, -4 \rangle$ (at an arbitrary point (x,y)).

Step 1:

Step 2:

Step 3:

Gradient

If $f(x,y)$ is differentiable, $\vec{u} = \langle u_1, u_2 \rangle$, $\|\vec{u}\| = 1$, then

$$D_{\vec{u}} f(x,y) = f_x(x,y) \cdot u_1 + f_y(x,y) \cdot u_2 \qquad (*)$$

<u>Def</u>. Let $f(x,y)$ be a function of two variables such that f_x and f_y exist. Then the vector

We can rewrite $(*)$ as

Examples

1. $f(x,y) = x^2 - xy + 3y^2$. Find $\nabla f(x,y)$.

2. $f(x,y) = \sin(3x)\cos(3y)$. Find $\nabla f(x,y)$

Gradient as the direction of the steepest ascent

Consider a function $f(x,y)$ and a point (x_0, y_0).
We know that $D_{\vec{u}} f(x_0, y_0)$ gives the rate of change of function f at point (x_0, y_0) in the direction $\vec{u}$.

Q: For which $\vec{u}$ is $D_{\vec{u}} f(x_0, y_0)$ the greatest?

In other words, which direction gives the greatest rate of change?

Suppose that f is differentiable.

Gradient as the direction of the steepest ascent

Recall that $-1 \leq \cos\varphi \leq 1$,

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Example

Find the direction for which the directional derivative of $f(x,y) = 2x^2 - xy + 3y^2$ at $(-2,3)$ is a maximum. What is the maximum value.

The direction of the most rapid increase :

The rate of change in this direction is