MATH 10C: Calculus III (Lecture B00)

mathweb.ucsd.edu/~ynemish/teaching/10c

Today: Local minima/maxima

Next: Strang 4.7

Week 7:

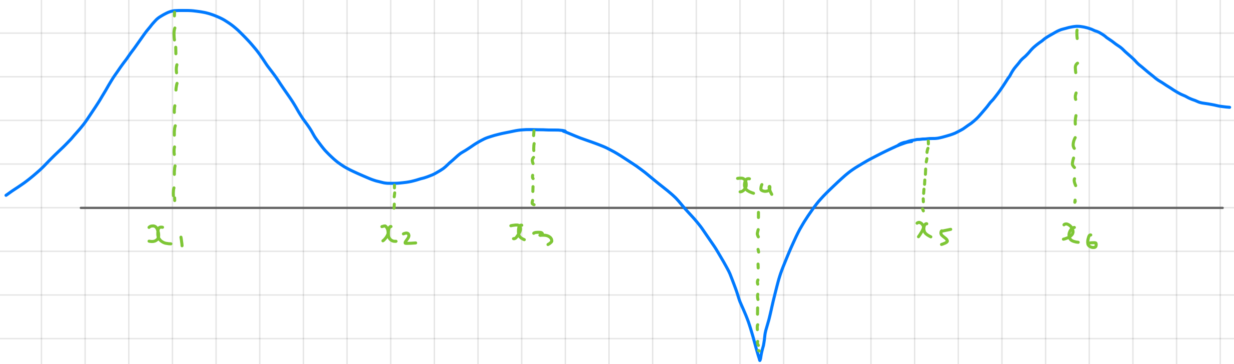
- homework 6 (due Friday, November 11)
- Midterm 2: Wednesday, November 16 (lectures 10-19)

Maxima and minima of functions of one variable

Let $f: \mathbb{R} \to \mathbb{R}$ be a function of one variable.

The point $x_0 \in \mathbb{R}$ is called a critical point of f if either $f'(x_0) = 0$ or $f'(x_0)$ does not exist.

Any local maximum or local minimum of f is a critical point.



Critical points of functions of two variables

Finding local minima/maxima in one dimension:

(i) identify critical points; (ii) determine which critical points are local minima/maxima.

We will extend this to functions of two variables. First, introduce the notion of a critical point for functions of two variables.

<u>Def</u>. Let $z = f(x,y)$ be a function of two variables defined at (x_0, y_0). Then (x_0, y_0) is called a

if either

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-

Critical points. Example

Find the critical points of the function

$$f(x,y) = \sqrt{4y^2 - 9x^2 + 24y + 36x + 36}$$

Start by computing f_x and f_y and finding (x,y) s.t.

$f_x(x,y) = 0$ and $f_y(x,y) = 0$ simultaneously

$f_x(x,y) =$

$f_y(x,y) =$

Next, find all (x,y) for which f_x or f_y does not exist:

all x,y s.t.

Critical points. Example (cont.)

Therefore, and

are possible critical points. We have to check that these points are in the domain of definition of f.

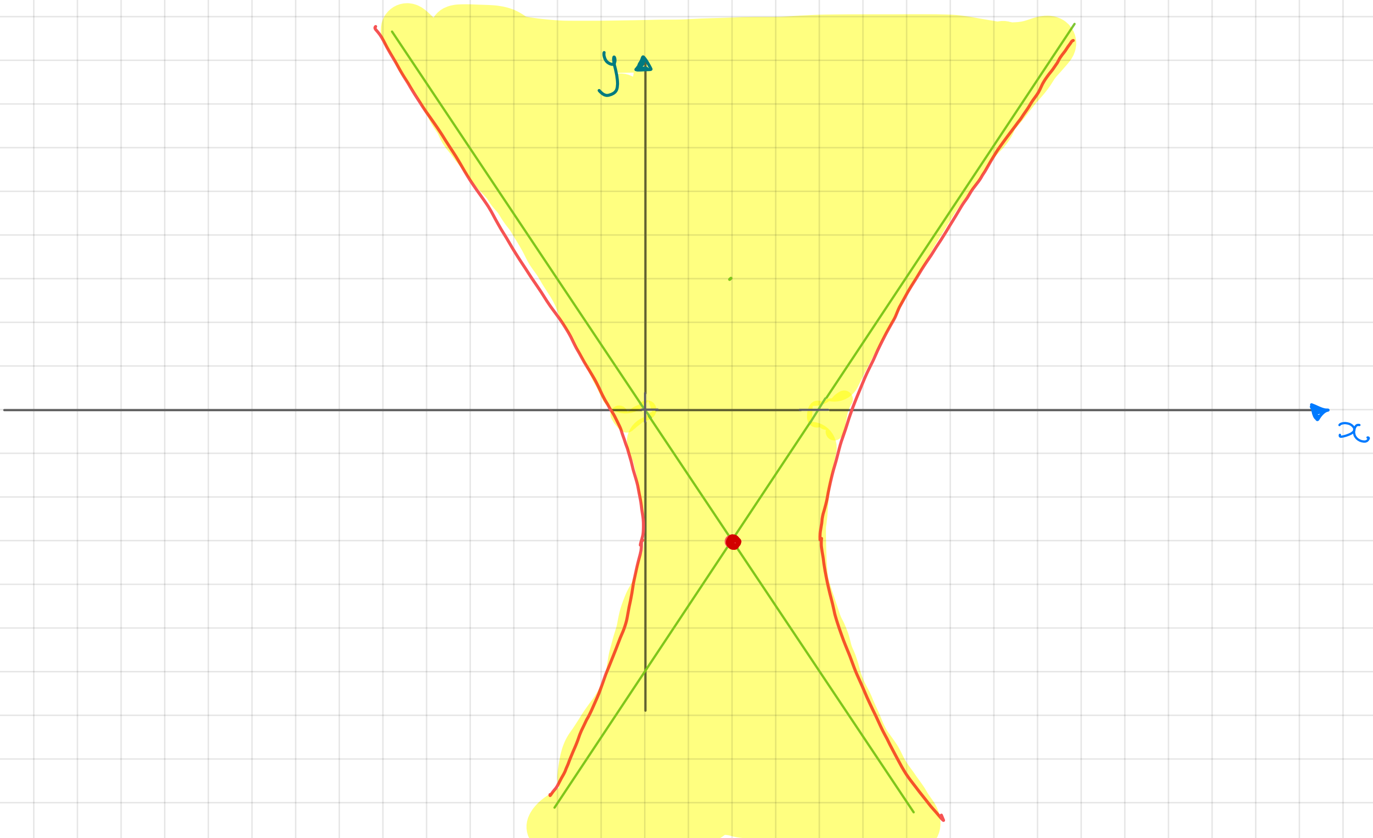
The domain of definition of f consists of all (x,y) s.t.

Clearly, all points satisfying $(*)$

Also, point $(2,-3)$ Therefore, the set of the critical points of f consists of and all points of the hyperbola

Critical points. Example (cont.)

Here is the plot of the domain of f and the critical points of f



Local minimum/maximum

Def Let $z = f(x, y)$ be a function of two variables.

Then f has if

for all points (x, y) within some disk

centered at (x_0, y_0). The number $f(x_0, y_0)$ is called

. If $(*)$ holds for all (x, y) in the domain

of f, we say that f has

Function f has a if

for all points (x, y) within some disk

centered at (x_0, y_0). The number $f(x_0, y_0)$ is called the

. If $(**)$ holds for all (x, y) in the domain

of f, we say that f has

Local minima and local maxima are called

Local extrema and critical points

Thm 4.16 Let $z = f(x,y)$ be a function of two variables,

Suppose

Example At the very top of a mountain the ground is flat. If there was slope in some direction, then you could go higher. Similarly, at the lowest point of a crater the ground is also flat ($\nabla f = 0$).

But the fact that the ground is flat ($\nabla f(x_0, y_0) = 0$)

that f has a local

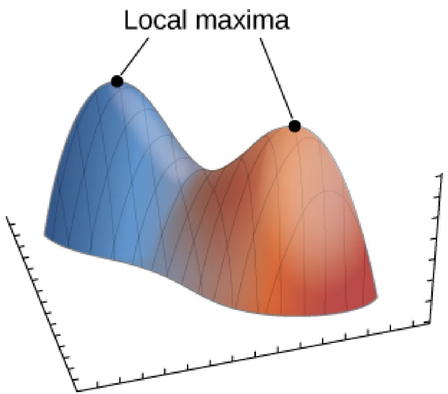
extremum at (x_0, y_0)

Saddle points

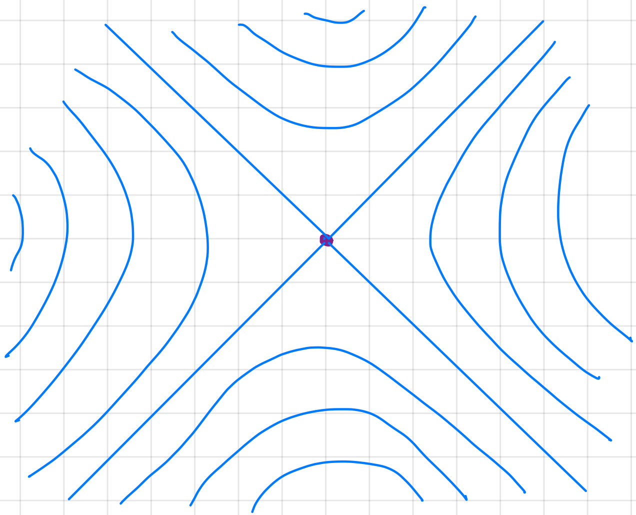
Def. Let $z = f(x,y)$ be a function of two variables.

We say that (x_0, y_0) is a if ,

, but f



Level curves around the saddle point have this shape



The second derivative test

Thm 4.17 (Second derivative test)

Suppose that $f(x,y)$ is a function of two variables for which the first- and second-order partial derivatives are continuous around (x_0, y_0). Suppose and . Define

(i) If and , then f has a

(ii) If and , then f has a

(iii) If , then f has a

(iv) If , then

Problem solving strategy

Problem:

Let $z = f(x, y)$ be a function of two variables for which the first- and second-ordered partial derivatives are continuous. Find local extrema.

Solution:

1. Determine critical points (x_0, y_0) where $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$
   Discard any points where f_x or f_y does not exist.
2. Caclulate D for each critical point
3. Apply the Second derivative test to determine if (x_0, y_0) is a local minimum, local maximum or a saddle point.