

Name (last, first): _____

Student ID: _____

Write your name and PID on the top of EVERY PAGE.

The exam consists of 16 questions. Each question has only one correct answer. Be sure to completely fill in the appropriate bubble in the bubble answer sheet.

DO NOT REMOVE ANY OF THE PAGES.

No calculators, phones, or other electronic devices are allowed.

You are allowed to use one 8.5 by 11 inch sheet of paper with handwritten notes (on both sides); no other notes (or books) are allowed.

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1. (5 points) Let O, P, Q, R be points in \mathbb{R}^3 with coordinates

$$O = (0, 0, 0), \quad P = (3, 2, -1), \quad Q = (0, -1, 2), \quad R = (6, 2, 0).$$

Which of the following vectors is parallel to the vector $\mathbf{v} = \left\langle -1, \frac{1}{3}, \frac{1}{2} \right\rangle$?

A. \overrightarrow{PQ}

B. \overrightarrow{OP}

C. \overrightarrow{OQ}

D. \overrightarrow{PR}

E.* None of the above

2. (5 points) Find a *unit* vector \mathbf{u} in the direction opposite of $\langle 2, -2, 1 \rangle$.

A. $\mathbf{u} = \left\langle \frac{-4}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{4}{\sqrt{3}} \right\rangle$

B.* $\mathbf{u} = \left\langle \frac{-2}{3}, \frac{2}{3}, \frac{-1}{3} \right\rangle$

C. $\mathbf{u} = \left\langle 1, \frac{1}{2}, -1 \right\rangle$

D. $\mathbf{u} = \left\langle \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \right\rangle$

E. None of the above

3. (5 points) Assume that $\mathbf{u} \cdot \mathbf{v} = -1$, $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 2$. What is the value of $(\mathbf{u} + \mathbf{v}) \cdot (\mathbf{u} + 3\mathbf{v})$?

A. -3

B.* 12

C. -6

D. 0

E. None of the above

4. (5 points) Compute the area of the triangle with vertices

$$P = (1, 1, 0), \quad Q = (2, 3, 0), \quad R = (2, 2, -1).$$

A. $\sqrt{3}$

B. -1

C. $\frac{1}{2}$

D.* $\frac{\sqrt{6}}{2}$

E. None of the above

5. (5 points) Find the equation of the plane which passes through point $P = (-1, 1, -1)$ and is parallel to $x + 2y + \frac{1}{2}z = -1$.
- A.* $-2x - 4y - z = -1$
B. $x + \frac{1}{2}y + 2z = -1$
C. $x + 2y + z = 0$
D. $-x - y - z = 1$
E. None of the above
6. (5 points) Find the distance from the point $(0, -1, 0)$ to the plane $-2x - y + 3z = 3$.
- A. $\frac{\sqrt{7}}{2}$
B. $\frac{2}{\sqrt{3}}$
C. $\frac{\sqrt{7}}{\sqrt{3}}$
D.* $\frac{2}{\sqrt{14}}$
E. None of the above
7. (5 points) Suppose $\mathbf{r}(t) = \langle e^{-t+3}, \cos(\pi t), 2t - 1 \rangle$ represent the position of a particle at time t , where the z -component represents the height of the particle. What is the velocity of the particle when its height is 5?
- A.* $\langle -1, 0, 2 \rangle$
B. $\langle 3e^{-3}, -\pi, 2 \rangle$
C. $\langle e^3, 0, 5 \rangle$
D. $\langle 1, \pi, 2 \rangle$
E. None of the above
8. (5 points) Find the parametric equation of the tangent line to $f(t) = \langle -t^3, 2t^3, 2t - 4 \rangle$ at $t_0 = 2$.
- A. $\langle -8, 16, 0 \rangle + t\langle 16, -32, -4 \rangle$
B. $\langle 16, 12, 1 \rangle + t\langle 16, 12, 1 \rangle$
C.* $\langle 16, -32, -4 \rangle + t\langle -12, 24, 2 \rangle$
D. $\langle 16, 8, 1 \rangle + t\langle -12, 24, 2 \rangle$
E. None of the above

9. (5 points) Let $f(x, y) = e^{x^2 - 2x + y^2}$. Compute the partial derivative $f_x(2, 0)$.
- A.* 2
 - B. $-2e^2$
 - C. e^2
 - D. 0
 - E. None of the above
10. (5 points) Let $f(x, y) = (x^2 + y)e^{2y}$. Compute $f_{xy}(1, 1)$.
- A.* $4e^2$
 - B. 2
 - C. $2e$
 - D. 0
 - E. None of the above
11. (5 points) Given $z = f(x, y)$, $x = x(u, v)$, $y = y(u, v)$, with $x(2, 1) = 1$ and $y(2, 1) = 2$, calculate $z_u(2, 1)$ using the values below
- $$\begin{aligned} f_x(2, 1) = c, \quad f_y(2, 1) = d, \quad f_x(1, 2) = p, \quad f_y(1, 2) = q, \\ x_u(2, 1) = 1, \quad x_v(2, 1) = 2, \quad y_u(2, 1) = 3, \quad y_v(2, 1) = -1. \end{aligned}$$
- A. $-p + q$
 - B. $3c + 2d$
 - C. $p + q + 2$
 - D.* $p + 3q$
 - E. None of the above
12. (5 points) Calculate the directional derivative of $f(x, y) = x^2y^2$ in the direction of $\mathbf{v} = \langle 2, 1 \rangle$ at the point $P = (1, -1)$. Remember to normalize the direction vector.
- A. $\frac{\sqrt{5}}{12}$
 - B.* $\frac{2}{\sqrt{5}}$
 - C. $-\frac{3}{\sqrt{5}}$
 - D. 0
 - E. None of the above

13. (5 points) Find the critical point of the function $f(x, y) = 3x^2 + 3y^2 + xy + 7x$.
- A. $(-1, 0)$
 - B. $(\frac{1}{2}, \frac{3}{2})$
 - C. $(1, 6)$
 - D.* $(-\frac{6}{5}, \frac{1}{5})$
 - E. None of the above
14. (5 points) The point $P = (-2, 0)$ is a critical point of the function $f(x, y) = x^3 - y^2 - 12x$. Use the second derivative test to determine if P is a point of local minimum, local maximum or a saddle point.
- A. Local minimum
 - B.* Local maximum
 - C. Saddle point
 - D. The second derivative test is inconclusive
 - E. None of the above
15. (5 points) Find the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint $4x + 2y = 5$.
- A. $\frac{10}{36}$
 - B.* $\frac{5}{4}$
 - C. $\frac{120}{25}$
 - D. Minimum does not exist
 - E. None of the above
16. (5 points) Find three positive real numbers whose sum is 3 and whose product is a maximum.
- A.* 1
 - B. $\frac{1}{3}$
 - C. 3
 - D. Maximum does not exist
 - E. None of the above

